Density of a Hot Jupiter

Last Thursday (February 1) in class, Professor Bailyn discussed a "Hot Jupiter" that is aligned in just the right way such that the planet periodically passes in front of the star (i.e. "transits"). We have observed another nearby sun-like star that also transits. In order to determine the composition of the star, you set out to measure the density. Since we know that density (ρ) = mass (M) / volume (V), we need to measure the mass and volume *of the planet* in order to get the density *of the planet*.

$v = \frac{2\pi a}{P}$	$a^3 = P^2 M$	$1 \text{ AU} = 1.5 \times 10^{11}$ m
$v_* M_* = v_p M_p$	$V = \frac{4}{3}\pi R^3$	$1\ M_\odot = 2 \times 10^{30}$ kg
$\rho = \frac{M}{V}$	$A = \pi R^2$	$1 \text{ year} = 3 \times 10^7$ s

Step 1: Mass

From Doppler measurements, it's been found that the reflex velocity of the star (due to the planet) is 85 m/s and that the period of the planet's orbit is 3.5 days ($= 10^{-2}$ yrs).

- What is the semi-major axis (a) of the planet's orbit?

$$\begin{aligned} a^3 &= P^2 M \\ &= (10^{-2})^2 * 1 \\ &= 10^{-4} \\ a &= (100 \times 10^{-6})^{1/3} \\ a &= 5 \times 10^{-2}\text{A.U.} = 7.5 \times 10^9\text{m} \end{aligned}$$

- What is the velocity of the planet in its orbit?

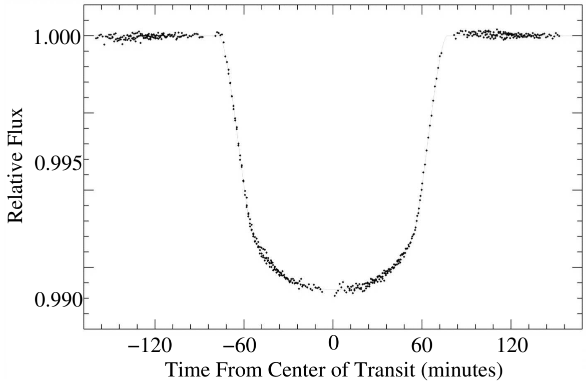
$$\begin{aligned} v &= \frac{2\pi a}{P} \\ &= \frac{6 \times 7.5 \times 10^9}{10^{-2} \times 3 \times 10^7}\ [\text{m/s}] \\ &= 1.5 \times 10^5 \text{ m/s} \end{aligned}$$

- What is the mass of the planet?

$$\begin{aligned} v_* M_* &= v_P M_P \\ 85 \text{ m/s} \times 2 \times 10^{30} \text{ kg} &= 1.5 \times 10^5 \text{ m/s } M_P \\ \frac{8.5 \times 10^1}{1.5 \times 10^5}\, 2 \times 10^{30} \text{ kg} &= M_P \\ 6 \times 10^{-4} \times 2 \times 10^{30} \text{ kg} &= M_P \\ M_P &= 1.2 \times 10^{27} \text{ kg} \end{aligned}$$

Step 2: Volume

In addition to the radial velocity measurements made above, you also observe the planet making transits across the star. These transits last for about 2.5 hours and, at the point of greatest eclipse, block out 1% of the light from the star.



- Assuming that the *radius of the star* is the same as that of the Sun (7×10^8 m), what is the *radius of the planet*?

  The fraction of light blocked by the planet is the ratio of the areas of the star and the planet:

$$
\begin{aligned}
\frac{A_P}{A_*} &= 0.01 \\
\frac{\pi R_P^2}{\pi R_*^2} &= 10^{-2} \\
\frac{R_P^2}{R_*^2} &= 10^{-2} \\
\frac{R_P}{R_*} &= (10^{-2})^{1/2} \\
\frac{R_P}{7 \times 10^8} &= 10^{-1} \\
R_P &= 7 \times 10^7 \text{ m}
\end{aligned}
$$

- What is the volume of the planet?

$$
\begin{aligned}
V &= \frac{4}{3}\pi R^3 \\
V &= 4R^3 \\
V &= 4 \times (7 \times 10^7)^3 \\
V &= 4 \times 350 \times 10^{21} \\
V &= 1.4 \times 10^{24} \text{ m}^3
\end{aligned}
$$

Step 3: Density

- What is the density of the planet?

$$\begin{aligned}\rho &= \frac{M}{V}\\ \rho &= \frac{1.2\times10^{27}\text{ kg}}{1.4\times10^{24}}\text{ m}^3\\ \rho &= 1\times10^{3} = 1000\text{ kg/m}^3\end{aligned}$$

- How does that density compare to that of water (1000 kg/m^3)?

  The density of this planet is roughly the same density as water.

- What does this density tell you about the composition of the planet?

  This tells you that the planet can *not* be a rocky planet and, in fact, must be a large gas giant.

BONUS: How could you have figured out the star's radius if that information were omitted from the problem?

Since we know the transit time (2.5 hours) and the velocity of the planet (1.5×10^5 m/s), we can figure out the radius of the star using those numbers.

$$\begin{aligned}v &= \frac{d}{t}\\ 1.5\times10^{5}\text{ m/s} &= \frac{d}{2.5\text{ hours}}\\ 1.5\times10^{5}\text{ m/s} &= \frac{d}{9\times10^{3}\text{ s}}\\ 1.4\times10^{9}\text{ m} &= d\end{aligned}$$

Note that this is the *diameter* of the star, since it is the total time it takes the planet to move from one side of the star to the other. Since the radius is just half the diameter, this means the radius is 7×10^8 m.