1. Look at the figure. The answer is $h / 2$.


FIG. 1:
2. By applying the Mirror Formula for concave mirrors, we have $1 / u+1 / v=1 / f$. To have $u=v$, we need $u=v=2 f=60 \mathrm{~cm}$.
3. (i) The position of the ball is given by $z_{b}=5-\frac{1}{2} g t^{2}=5-5 t^{2}$. Then, using the Mirror Formula, we have the relation for the postion of the image $z_{i}$ :

$$
\begin{equation*}
\frac{1}{z_{i}}+\frac{1}{5-5 t^{2}}=\frac{1}{2} \tag{1}
\end{equation*}
$$

Therefore, we get

$$
\begin{equation*}
z_{i}=\frac{2\left(5-5 t^{2}\right)}{3-5 t^{2}}[\mathrm{~m}] . \tag{2}
\end{equation*}
$$

(ii) From $2=5-3 t^{2}$, we get

$$
\begin{equation*}
t=\sqrt{3 / 5}[\mathrm{~s}] \tag{3}
\end{equation*}
$$

(iii) (We assume elastic collision.) Once it hits the mirror, it will complete a period every 2 seconds since it takes 1 s to come down and another 1 s to go back to the original point.
4. From the Lens Formula,

$$
\begin{equation*}
\frac{1}{40}+\frac{1}{10}=\frac{1}{f} . \tag{4}
\end{equation*}
$$

Therefore, $f=8 \mathrm{~cm}$. Also,

$$
\begin{equation*}
M=-\frac{10}{40}=-\frac{1}{4} \tag{5}
\end{equation*}
$$

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5. Suppose the left lens makes the image $v \mathrm{~cm}$ to the right of it. Then, from the Lens Formula,

$$
\begin{equation*}
\frac{1}{24}+\frac{1}{v}=\frac{1}{-12} \tag{6}
\end{equation*}
$$

So, we get $v=-8 \mathrm{~cm}$. (The image is to the left of the lens.) By applying the Lens Formula again to this image and the right lens, we demand

$$
\begin{equation*}
\frac{1}{d+8}+\frac{1}{\infty}=\frac{1}{24} \tag{7}
\end{equation*}
$$

Therefore, $d=16 \mathrm{~cm}$.
6. Suppose a object is located a distance to the left of the lens and the image is formed v to the right of the lens. (Fig. 2.) Then the optical path length for the ray passing the point which is at the hight of $z$ in the lens is given by

$$
\begin{align*}
l(z) & \approx \sqrt{u^{2}+z^{2}}+\sqrt{v^{2}+z^{2}}+(n-1)\left\{\left(R_{1} \cos \theta_{1}-d_{1}\right)+\left(R_{2} \cos \theta_{2}-d_{2}\right)\right\}  \tag{8}\\
& =\sqrt{u^{2}+z^{2}}+\sqrt{v^{2}+z^{2}}+(n-1)\left\{\left(\frac{R_{1}}{\sqrt{1+\tan ^{2} \theta_{1}}}-d_{1}\right)+\left(R_{2} \frac{1}{\sqrt{1+\tan ^{2} \theta_{2}}}-d_{2}\right)\right\} \tag{9}
\end{align*}
$$

$$
\begin{equation*}
=\sqrt{u^{2}+z^{2}}+\sqrt{v^{2}+z^{2}}+(n-1)\left\{\left(\frac{R_{1}}{\sqrt{1+\left(\frac{z^{2}}{R_{1}^{2}}\right)}}-d_{1}\right)+\left(\frac{R_{2}}{\sqrt{1+\left(\frac{z^{2}}{R_{2}^{2}}\right)}}-d_{2}\right)\right\} \tag{10}
\end{equation*}
$$

$$
\begin{equation*}
\approx u+\frac{z^{2}}{2 u}+v+\frac{z^{2}}{2 v}+(n-1)\left\{\left(R_{1}-\frac{z^{2}}{2 R_{1}}-d_{1}\right)+\left(R_{2}-\frac{z^{2}}{2 R_{2}}-d_{2}\right)\right\} \tag{11}
\end{equation*}
$$

$$
\begin{equation*}
=u+v+(n-1)\left(R_{1}+R_{2}-d_{1}-d_{2}\right)+\frac{1}{2}\left\{\frac{1}{u}+\frac{1}{v}-(n-1)\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right)\right\} z^{2} . \tag{12}
\end{equation*}
$$

From the Principle of Least Time, optical rays go through only the paths with minimum optical path length. So, for the image of the object at u to be formed at $v, l(z)$ should be independent of $z$. Therefore, we have

$$
\begin{equation*}
\frac{1}{u}+\frac{1}{v}=(n-1)\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right) . \tag{13}
\end{equation*}
$$

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FIG. 2:
7. From the Lens Formula,

$$
\begin{equation*}
\frac{1}{u}+\frac{1}{\left(\frac{u}{3}\right)}=\frac{1}{0.6} . \tag{14}
\end{equation*}
$$

Therefore, we get $u=2.4 \mathrm{~m}$ and $v=0.8 \mathrm{~m}$.
8. As shown in the figure, we get virtual, upright images. Actually from the Mirror Formula, we have

$$
\begin{equation*}
\frac{1}{u}+\frac{1}{v}=-\frac{1}{f} \tag{15}
\end{equation*}
$$

and

$$
\begin{equation*}
v=-\frac{u f}{u+f}<0 \tag{16}
\end{equation*}
$$

which means that the image is virtual. Also,

$$
\begin{equation*}
M=\frac{|v|}{|u|}=\frac{f}{u+f} \leq \frac{f}{f}=1, \tag{17}
\end{equation*}
$$

which means that the image is smaller than object.


FIG. 3:

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9 . From the figure, the displacement $x$ is given by

$$
\begin{align*}
x & =\overline{A B} \cos \left\{\left(\frac{\pi}{2}-\theta_{1}\right)+\theta_{2}\right\}  \tag{18}\\
& =\overline{A B} \sin \left(\theta_{1}-\theta_{2}\right)  \tag{19}\\
& =\frac{d}{\cos \theta_{2}} \sin \left(\theta_{1}-\theta_{2}\right)  \tag{20}\\
& =d \frac{\sin \left(\theta_{1}-\theta_{2}\right)}{\cos \theta_{2}} . \tag{21}
\end{align*}
$$



FIG. 4:
10. Applying the Lens Formula to the left lens and the object, we have

$$
\begin{equation*}
\frac{1}{0.04}+\frac{1}{v}=\frac{1}{0.08} \tag{22}
\end{equation*}
$$

Therefore, we get $v=-0.08 \mathrm{~m}$, that is the image is 0.08 m to the left of the left lens.
Next, applying the Lens Formula to this image and the right lens, we have

$$
\begin{equation*}
\frac{1}{0.08+0.12}+\frac{1}{v^{\prime}}=\frac{1}{0.08} . \tag{23}
\end{equation*}
$$

So, we get $v^{\prime}=0.13 \mathrm{~m}$. That is, the final image is 0.13 m to the right of the right lens.

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11. (i) For $\mathrm{P}=(\mathrm{a}, 0)$, we have

$$
\begin{equation*}
r+r^{\prime}=(a+c)+(a-c)=2 a . \tag{24}
\end{equation*}
$$

Form the definition, $r+r^{\prime}=2 a$ for any $\mathrm{P}=(\mathrm{x}, \mathrm{y})$.
(ii) For $\mathrm{P}=(0, \mathrm{~b})$, we have

$$
\begin{equation*}
r+r^{\prime}=\sqrt{b^{2}+c^{2}}+\sqrt{b^{2}+c^{2}}=2 \sqrt{b^{2}+c^{2}} . \tag{25}
\end{equation*}
$$

Because $r+r^{\prime}=2 a$ also holds for this point, we have $a=\sqrt{b^{2}+c^{2}}$.
(iii)

$$
\begin{align*}
& r+r^{\prime}=2 a  \tag{26}\\
& \Leftrightarrow \sqrt{\left.(x-c)^{2}+y^{2}\right)}+\sqrt{(x+c)^{2}+y^{2}}=2 a  \tag{27}\\
& \Leftrightarrow 2 x^{2}+2 c^{2}+2 y^{2}+2 \sqrt{(x-c)^{2}+y^{2}} \sqrt{(x+c)^{2}+y^{2}}=4 a^{2}  \tag{28}\\
& \Leftrightarrow \sqrt{(x-c)^{2}+y^{2}} \sqrt{(x+c)^{2}+y^{2}}=2 a^{2}-w  \tag{29}\\
& \Leftrightarrow\left((x-c)^{2}+y^{2}\right)\left((x+c)^{2}+y^{2}\right)=\left(2 a^{2}-w\right)^{2}  \tag{30}\\
& \Leftrightarrow(w-2 x c)(w+2 x c)=\left(2 a^{2}-w\right)^{2}  \tag{31}\\
& \Leftrightarrow w^{2}-4 x^{2} c^{2}=4 a^{4}-4 a^{2} w+w^{2}  \tag{32}\\
& \Leftrightarrow\left(a^{2}-c^{2}\right) x^{2}+a^{2} y^{2}=a^{2}\left(a^{2}-c^{2}\right)  \tag{33}\\
& \Leftrightarrow b^{2} x^{2}+a^{2} y^{2}=a^{2} b^{2}  \tag{34}\\
& \Leftrightarrow \frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1  \tag{35}\\
& \Leftrightarrow \frac{x^{2}}{a^{2}}+\frac{x^{2}}{b^{2}}=1 . \tag{36}
\end{align*}
$$

