① a $M = -19$

$d = 10^9$ lightyears $= 3 \times 10^8$ pc
(where 1 ly $= 3.3$ pc)

$$m - M = 5 \log\left(\frac{3 \times 10^8}{10}\right) = 5 \log(3 \times 10^7) = 5(\log 3 + \log 7)$$
$$= 5(0.5 + 7) = 37.5$$

$m - M = 37.5$

$m - (-19) = 37.5$

$\boxed{m = 18.5 \text{ magnitudes}}$

b $\lambda_0 = 0.65$ microns

$$z = \frac{1}{a} - 1 = \frac{1}{0.9} - 1 = 1.11 - 1 = 0.11$$

$$z = \frac{\lambda - \lambda_0}{\lambda_0}$$

$$0.11 \lambda_0 = \lambda - \lambda_0$$
$$0.11 \lambda_0 + \lambda_0 = \lambda$$
$$\lambda_0 (1.11) = \lambda$$
$$\lambda = 0.65 (1.11) = \boxed{0.72 \text{ microns}}$$

OR

$$\frac{a_{now}}{a_{then}} = \frac{\lambda_{obs}}{\lambda_{emit}}$$

$$\frac{1}{0.9} = \frac{\lambda_{obs}}{0.65 \text{ microns}}$$

$$\boxed{\lambda_{obs} = 0.72 \text{ microns}}$$

II $z = 1.5$

$m - M = 42.5$

$a = \frac{1}{1+z} = \frac{1}{1+1.5} = 0.4$

$m - M = 5 \log\left(\frac{d}{10}\right)$

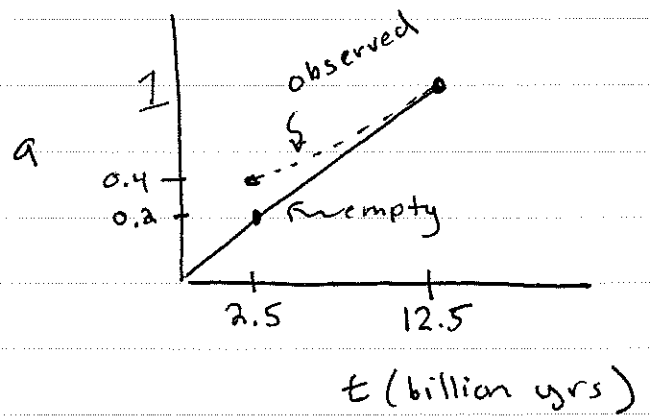
$42.5 = 5 \log\left(d/10\right)$

$d = 3 \times 10^{9}\ pc = 10^{10}$ lightyears

point "now" $a = 1$ $t = 12.5$ billion yrs

point "then" $a = 0.4$ $t = 2.5$ billion yrs

↑ or 10^{10} billion yrs IN THE PAST

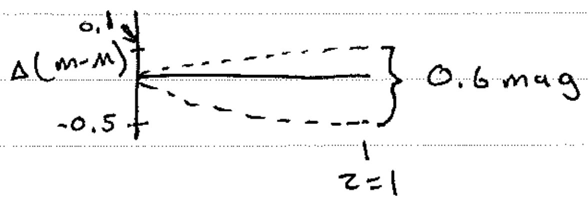


universe is <u>accelerating</u>

we know this because at $t = 2.5$ billion years the expansion of the empty universe had a rate of $z = 4$ ($a = 0.2$ at 2.5 billion on the empty line), while the observed point has a slower expansion at the same time, $z = 1.5$. Since all points must meet at "now" the observed universe expansion needs to speed up to end up matching the empty universe "now".

(III) [a] We need dark energy (DE) to explain <u>why</u> the observations of SNe Ia appear fainter at high z than would be expected in a pure $\Omega_M = 1$ universe. This assumes that Ia's are STANDARD CANDLES! If distance SNe Ia's are fainter than nearby ones, than the fact that high z Ia's <u>appear</u> faint may be due merely to the fact that they <u>ARE</u> faint. In that case, DE need not be invoked to explain the observed points.

[b] from $\Delta(m-M)$ plot at $z=1$ the magnitude difference between the $\Omega_M = 0.25$ $\Omega_\Lambda = 0.75$ line and the $\Omega_M = 1$ line is 0.6 magnitudes



$$0.6 = 2.5 \log\left(\frac{b_{z=1}}{b_{z=0}}\right)$$

$$\boxed{b_{z=1} = 0.58\, b_{z=0}}$$

$\Rightarrow$ nearby SNe are about twice as bright as the distance SNe