

# Astronomy 160: Frontiers and Controversies in Astrophysics

## Homework Set # 7 Solutions

- 1) Suppose the density of the universe is exactly equal to the critical density (with  $H = 70\text{km/s/Mpc}$ ), and all the matter is contained within identical galaxies, one per cubic megaparsec.

a) Determine the mass of each galaxy.

$$\begin{aligned}\rho = \rho_c &= \frac{3H^2}{8\pi G} = \frac{H^2}{8G} = \frac{(70)^2}{8 \times 7 \times 10^{-11}} \frac{[km \cdot s^{-1} \cdot Mpc^{-1}]}{[m^3 \cdot kg^{-1} \cdot s^{-2}]} \\ &= \frac{50 \times 10^2 \times 10^6}{50 \times 10^{-11}} \frac{[kg]}{[m \cdot Mpc^2]} = \frac{10^{19}}{3 \times 10^{-23}} \frac{[kg]}{[Mpc^3]} \\ \rho &= \frac{1}{3} 10^{42} = 3 \times 10^{41} \text{ kg/Mpc}^3\end{aligned}$$

Since there is only one galaxy per cubic Mpc, the mass of each galaxy is  $3 \times 10^{41}$  kg.

- b) Most stars are a little bit less massive than the Sun, but a lot less bright. Suppose an average solar mass of material in these galaxies generates only 1/10 the brightness of the Sun, calculate the apparent magnitude of the nearest such galaxy (at a distance of 1 Mpc).

The total number of solar mass “chunks” is:

$$N = \frac{3 \times 10^{41}}{2 \times 10^{30}} = 1.5 \times 10^{11}$$

So the total brightness of the galaxy is:

$$b_g = 1.5 \times 10^{11} \times \frac{b_\odot}{10} = 1.5 \times 10^{10} b_\odot$$

The absolute magnitude of the nearest galaxy is given by:

$$M_g - M_\odot = -2.5 \log(b_g/b_\odot)$$

$$M_g = M_\odot - 2.5 \log(1.5 \times 10^{10} b_\odot/b_\odot)$$

$$M_g \approx M_\odot - 2.5 \log(10^{10}) = M_\odot - 2.5 \times 10 = 5 - 25 = -20$$

The apparent magnitude is then:

$$m_g - M_g = 5 \log(D/10pc)$$

$$m_g = -20 + 5 \log(10^6/10) = -20 + 5 \log(10^5) = -20 + 5 \times 5$$

$$m_g = -20 + 25 = 5$$

- c) **Suppose you observe a star orbiting around one of these galaxies, 30 kiloparsecs away from the center of the galaxy. How fast would it be moving (for purposes of this problem you can assume that essentially *all* the mass of the galaxy is contained within 30 kiloparsecs of the center).**

The velocity is given by:

$$V^2 = \frac{GM}{a} = \frac{7 \times 10^{-11} \cdot 3 \times 10^{41}}{30 \times 10^3 \cdot 3 \times 10^{16}} = 2 \times 10^{10} [m^2/s^2]$$

$$V = (2 \times 10^{10})^{1/2} = 1.5 \times 10^5 m/s$$

$$V = 150 km/s$$

- 2) a) **Explain why the claim that the high redshifts of quasars are *not* due to the expansion of the Universe undermines support for the Big Bang.**

If the high redshifts of quasars are *not* due to the expansion of the universe, this means that these objects are not necessarily at large distances. One of the primary claims that Big Bang theory makes is that the universe is different in the past than it is today. The prevalence of quasars at large redshifts (large distances in the orthodox interpretation) and the dearth of quasars locally is often presented as evidence that **the universe changes in time**, a key component of Big Bang Cosmology. Note that Arp's interpretation of quasar redshifts (i.e. some intrinsic property) does *not* necessarily rule out the expansion of the universe; it just means that the quasars are *local* objects. In fact, even the steady state theory proposes that the universe is expanding.

- b) **Suggest a way (or ways) in which Arp's claims could be tested. (Extra credit for pointing to one or more specific examples of tests that have been carried out).**

There are many ways to test Arp's theories. One of the most straightforward is to do an All-Sky survey and look for quasars. If you find that quasars are not systematically located near other galaxies, this would demonstrate that Arp was incorrect. As several people correctly pointed out, one would need to correct for the fact that gravitational lensing of quasars by foreground galaxies makes the quasars brighter and, therefore more easily detectable.