1. The earth and sun are 8.3 light-minutes apart. Ignore their relative motion for this problem and assume they live in a single inertial frame, the Earth-Sun frame. Events A and B occur at $t = 0$ on the earth and at 2 minutes on the sun respectively. Find the time difference between the events according to an observer moving at $u = .8c$ from Earth to Sun. Repeat if observer is moving in the opposite direction at $u = .8c$.

2. Return to the Earth-Sun case above. (a) What is the speed of a spacecraft that makes the trip from the Sun to the Earth in 5 minutes according to the on board clocks? (b) What is the trip time in the Earth-Sun frame? (c) Find the square of the space-time interval between them in light-seconds.

   (You may need to come back to part (c) after I do space-time intervals in class. Do not just jump in and use some formula. Think in terms of events, assign as many possible space-time coordinates as you can to each event in any frame and use the LT. Measure time in minutes, distance in light-minutes. Imagine a rod going from earth to the sun, if that helps.

   Note that the spatial coordinate difference between events in spacecraft frame are not the same as distance between Earth and Sun in that frame. Even pre-Einstein, if I sit in my car going at 60 mph, I leave New Haven at t=0 (Event 1) and arrive at Boston at t= 2hrs (Event 2), the two events have the same coordinate in my frame (i.e., where I am in the car) $\Delta x = 0$, but that is not the distance between these towns.)

3. A muon has lifetime of $2 \cdot 10^{-6}s$ in its rest frame. It is created 100 km above the earth and moves towards it at a speed $2.97 \cdot 10^{8} m/s$. At what altitude does it decay? According to the muon, how far did it travel in its brief life?

4. An observer S who lives on the x-axis sees a flash of red light at $x = 1210m$. Then, after $4.96 \mu s$, a flash of blue at $x = 480m$. Use subscripts R and B to label the coordinates of the events.
   (i) What is the velocity relative to S of an observer S' who records the events as occurring at the same place?
   (ii) Which event occurs first according to S' and what is the measured time interval between these flashes? For the former you do not need to do a calculation. For the latter I suggest using the space-time interval.

5. Two rockets of rest length L_0 are approaching the earth from opposite directions at velocities $\pm c/2$. How long does one of them appear to the other?

6. A body quadruples its momentum when its speed doubles. What was the initial speed in units of c, i.e., what was u/c?

7. A body of rest mass m_0 moving at speed v collides with and sticks to an identical body at rest. What is the mass and momentum of the final clump?

8. A body of rest mass m_0 moving at speed v approaches an identical body at rest. Find V, the speed of a frame in which the total momentum is zero. Do this first by the law of composition of velocities starting with how you would do this non-relativistically. Next, repeat using the transformation law of for the components of the Energy-momentum vector. (You may need to come back to the second part after I do energy-momentum vector in class.)

9. Show that a photon cannot break up into an electron and a positron. For our purposes the electron and positron are identical particles with four-momenta (with $c = 1$) $P_1 = m_0(\gamma_1, \ \gamma_1 \mathbf{v}_1)$ and $P_2 = m_0(\gamma_2, \ \gamma_2 \mathbf{v}_2)$ where $\gamma = 1/\sqrt{1 - v^2}$. The photon four-momentum is $K = (\omega, \mathbf{k})$ with $\omega = |\mathbf{k}|$. I suggest you use energy-momentum conservation and the dot products of four vectors to show that this process is kinematically forbidden.

10. **In this problem let** $c = 1$. A particle of rest mass m_0 decays into a photon and loses rest mass δ in the bargain. Show that the photon energy is $\nu = \delta(1 - \frac{\delta}{2m_0})$. Note that the photon has zero rest mass, that is, the square of its four momentum vanishes. The problem can be solved most efficiently if you use four vectors and dot products between them, but I do not insist you do. Denote the four vector of the photon by $K = (\nu, \mathbf{k})$ and that of the particle before and after by $P = (E, \mathbf{p})$ and $P' = (E', \mathbf{p}')$.

TWO ADDITIONAL PROBLEMS

11. Consider a particle of rest mass m_0 moving at velocity v in your frame S. Write down expression for the components of its energy momentum vector $P = (p_0, p_1)$ in terms of m_0, v. Now see this particle from a frame S' moving at velocity u. What will be its velocity w and what will be the components of $P' = (p_0^1, p_1')$, first in terms of w and then in terms of w written in terms of u and v? Show that the primed coordinates are related to unprimed ones by the same Lorentz Transformation that relates x_0, x_1 to (x_0^1, x_1').

12. **OPTIONAL, SOLUTIONS WILL BE POSTED** Consider two rockets A and B of rest length $L_0 = 1 \ m$ travelling towards each other with a tiny shift in the y-direction so they do not collide, as in part (i) of Figure 1. Each sees the other approach it with speed u. According to A, when the tail of B passed the tip of A, a missile was fired from the tail of A towards B as in part (ii) of the figure. It will clearly miss due to length contraction of B as seen by A. But B will see the event as in part (ii) of the Figure and expect a hit. Who is right? (Ignore the time it takes the missile to hit its target).

It will be most educational to assign space-time coordinates to five events in each frame: tip of A passes tip of B (set it to (0,0) for both), tail of B passes tip of A, missile is fired, tip of B passes tail of A and finally tails pass. It will be useful to know that if an event occurs at either end of either rockets its spatial coordinates are no-brainers *in that rocket frame* since the tips are always at $x = 0, x' = 0$ and the

tails are at $x = -1$ and $x' = +1$. It should also be easy to find time elapsed between tip of your rocket passing my tip and my tail since I am of unit length and you are moving at speed u. Ditto for tail. As a check of your coordinates you may want to see that the space-time interval between any event and $(0,0)$ comes out same for both.
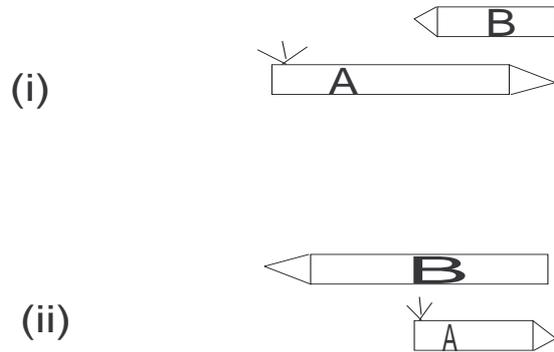
(i)

(ii)

FIG. 1. The point of view of A is in (i). We think B will see it as in (ii). But something is wrong since there was no hit as per A. The three funny lines coming transversely out of the tip of A are supposed to be the missile aimed at B.