Problem Set VI- Assign October 9, 2006 Due October 16.
Fall 2006 Physics 200a
Figures at the end
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Many problems ask for answers in terms of symbols and not numbers. You may need to invoke things like g in the answer even if I did not explicitly define them.

1. (i) What is the moment of inertia I_{CM} of a propeller with three blades (treated as rods) of mass m, length L, at 120^0 relative to each other? (ii) If a torque τ acts on this how long will it take to reach an angular velocity ω ? (iii) How many revolutions will it have made before reaching this ω? (iv) Get the numerical answers if $L = 1.25m$, $m = 12kg$, $\tau = 3000N \cdot m$, $\omega = 2000rads/s$.

2. Consider I for a rectangle of sides a (along the y-axis) and b (along the x-axis) about the two symmetry axes. (Rotate the rectangle about one of these axes and think of it as composed o of rods.) Show that about the axis parallel to x, $I = \frac{1}{12}Ma^2$. Going back to the very definition of I, show that if this rectangle is spun around an axis through its CM and perpendicular to its area the moment of inertia will be $I = \frac{1}{12}M(a^2 + b^2)$.

3. A 4.8 kg block is resting at the top of a 30^o slope of height $1m$. It is attached to a cylindrical pulley of mass 1.7 kg and radius 8 cm by a massless string that unwinds as the block slides downhill. If the acceleration of the block is 1.9 m/s^2 what is μ_k? Find the velocity at the bottom of the slope using forces and torques. Repeat using energy ideas. See Figure(1).

4. Argue that that $\mathbf{A} \cdot (\mathbf{A} \times \mathbf{B}) = 0$. In three dimensions find the expression of $\mathbf{A} \times \mathbf{B}$ in terms of vector components and $\mathbf{i}$,$\mathbf{j}$ and $\mathbf{k}$.

5. A disk of radius R and mass M is spinning at an angular velocity ω_0 rad/s. A non-rotating concentric disk of mass m and radius r drops on it from a negligible

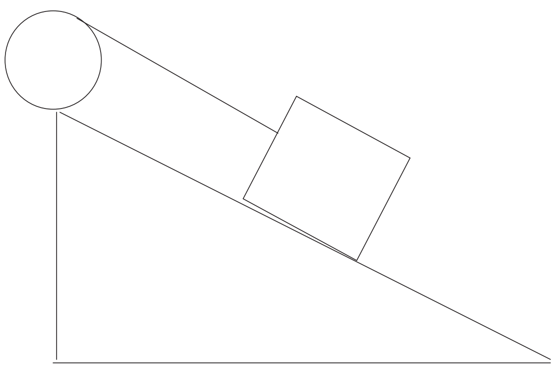

FIG. 1. The block started at the height h.

height and the two rotate together. (See Figure (2). Find the final ω and fraction of initial kinetic energy left.

6. A gyro consists of a solid disk of radius R mounted at one end of a shaft of zero mass and length l, the other end of which is on a pivot. The disk spins at ω rad/s and the gyro precesses at ω_p rad/s. What is l in terms of ω_p, ω, g and R? Give a number for l when $R = 6m$, $\omega_p = 2.6 rad/s$ and the disc is spinning at 450 rpm.

7. Two beads of mass m are free to slide on a rod of length l and mass M as in Figure(3). Initially the beads are at the center and the rod is spinning freely (with no external torque) at ω_0 rad/s about a vertical axis through its center. Slowly the beads move radially out (at negligible velocity). (i) Find $\omega(r)$, the angular velocity when the beads are r m from the center. (ii) What is ω when they just fly tangentially off the rod? Argue that ω does not change hereafter. (Hint: Using the formula $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ for the angular momentum of beads show that their $\mathbf{L}$'s do not change even though $\mathbf{r}$'s do. Also are there any torques on the spinning rod after the beads detach?) (iii) Why was the force of friction between beads and rod unimportant in the preceding discussion?

8. A sphere of radius R is supported by a rope attached to a wall as shown in Figure(4). The rope makes an angle θ with respect to the wall. The point where the rope is attached to the ball is such that if the line of the rope is extended it crosses the horizontal line through the center of the ball at a distance $3R/2$ from the wall. Show that the minimum μ_s between wall and ball for this to be possible is $\mu_s = \frac{1}{2}\cot\theta$. Evaluate this for $\theta = 30^0$. Hint: Find the right place to take torques. The usual suspects will not do.

9. A horizontal rod of mass 8 kg and length $2.4m$ is hinged to a wall and supported by a cable that makes an angle of 25^0 as shown in Figure (5). What is the tension T on the cable and what is the force exerted by the pivot? Repeat if in addition a 25 kg weight is suspended at the end of the rod.

10. A ladder of length $6m$ mass 15 kg leans against a wall at angle 30^o *with respect to the wall*. With respect to the ground it has $\mu_s = .4$. How high can a 70 kg man climb before the ladder slips?

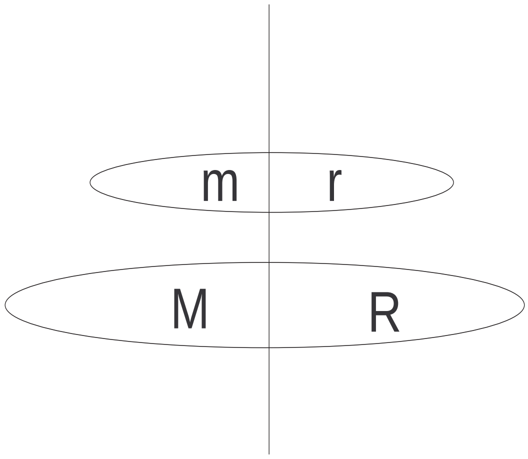

FIG. 2. The upper disk, initially at rest, falls with negligible speed on the lower one which is spinning. Their centers coincide.

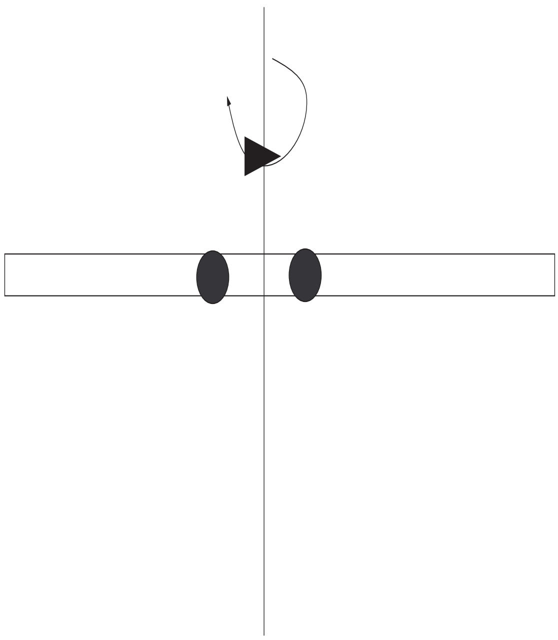

FIG. 3. The point-like beads can slide along the rod which is spinning around a vertical axis through its CM. The beads start out at the center of the rod.

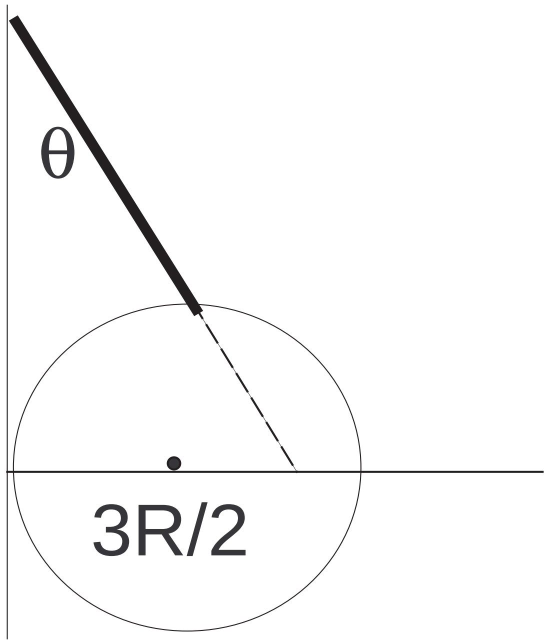

FIG. 4. The rope is attached to the wall at one end and the ball at the other. If extrapolated, line of rope crosses the horizontal line via center a distance $3R/2$ from wall.

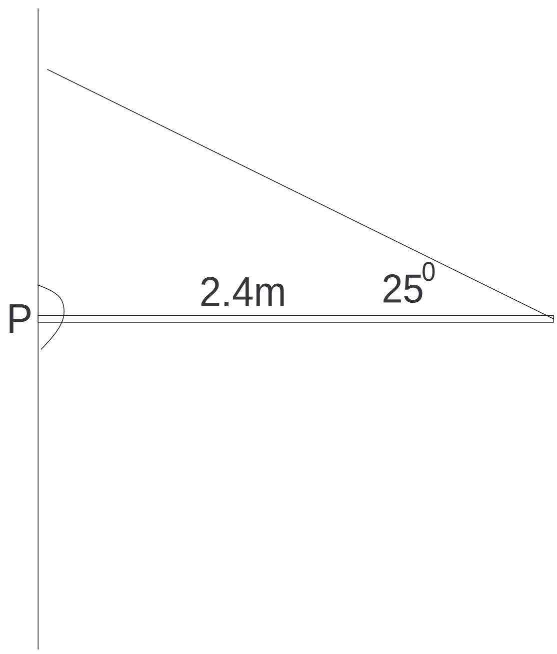

FIG. 5. The rope is attached to the wall at one end and the rod at the other. P is the pivot.