Practice Problems on Asymmetric Information

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These questions are designed to give you some practice applying ideas to do with asymmetric information. The problems are optional, for practice only, not for handing in. Most of the third problem is quite hard. The fourth problem is about auctions. I will supply answers on the web toward the end of next week.

(1) Entry Deterrence (Harbaugh). Consider a two-period game. In the first period, an incumbent monopolist sets the price for its product. In the second period a potential entrant might enter the market or might stay out. The incumbent is either low cost or high cost. The entrant does not know which: the incumbent could be low or high cost with equal probability. In the first period, if the incumbent chooses to have a high price, it earns 200 if it is low cost or 100 if it is high cost. If it chooses a low price, it earns 150 if it is low cost or 0 if it is high cost. While it hurts profits in the first period, setting a low price might discourage entry and boost profits in the second period. In the second period, if there is no entry, the incumbent will choose the high price and get 200 or 100 depending on its type. If there is entry, however, competition will force the incumbent to get only 50 if it is the low-cost type and only 10 if it is the high-cost type. The entrant must make an unrecoverable investment if it decides to enter the market. Its profits are -25 if the incumbent is low cost and 50 if the incumbent is high cost. The incumbent first decides the first-period price, then the entrant makes her decision.

  (a) Consider a putative *separating* equilibrium in which the incumbent would charge the low price in the first period if it is the low-cost type and would charge the high price if it were the high-cost type. Show that this is in fact an equilibrium. In particular, what does the potential entrant believe about the incumbent when seeing each price, and does it enter; and could either type of incumbent do better by deviating in the first period.

  (b) Consider a perverse putative *separating* equilibrium in which the incumbent would charge the high price in the first period if it is the low-cost type and would charge the low price if it were the **high** cost type. Is this an equilibrium? Explain.

(2) In the News. Hillary is considering running for president. A private opinion poll has told her whether she would have a *high* or *low* chance of winning the general election assuming she is her party's candidate. She has no way to communicate the results of this poll verifiably to other potential candidates: polling numbers are too easy to fabricate. After observing the poll, however, Hillary can choose whether or not to write another book with her husband. Regardless of Hillary's chance of winning, writing the book would cost Hillary 1 util. Think of this as loss

of self-esteem.

Al gets to decide whether or not to enter the primary after observing whether or not Hillary writes the book. Hillary would prefer Al to stay out of the primary race. Al staying out is worth 4 utils to Hillary if Hillary would have a *high* chance of winning the general election; and Al staying out is worth 2 utils to Hillary if Hillary would have a *low* chance of winning the general election. If Al runs, Hillary gains 0 utils regardless of his chances.

Al only wants to run if Hillary's chances in the general election are bad. Al gets 2 utils from running if Hillary would have a *low* chance of winning the general election; and Al gets -1 utils from running if Hillary would have a *high* chance of winning the general election. If Al stays out then his payoff is 0. Initially, before seeing whether or not Hillary writes a book, Al thinks Hillary is equally likely to have a *low* or *high* chance of winning the general election.

The following table may be helpful.

	high-chance Hillary	low-chance Hillary
Hillary's book cost	1	1
Hillary's gain if Al stays out	4	2
Hillary's gain if Al runs	0	0
Al's payoff if he runs	-1	2
Al's payoff if he stays out	0	0
Al's initial belief	$\frac{1}{2}$	$\frac{1}{2}$

(a) Is there a separating pure-strategy equilibrium in which Hillary makes a different decision whether or not to write the book depending on whether she has a *high* or *low* chance of winning the general election. If such equilibria exist, describe one and show it is an equilibrium. If no such equilibrium exists, explain why not.

Now suppose that Hillary can still write the book or do nothing, but she also has a third choice. If she does not write the book, she can go on Saturday Night Live with her husband. This is likely to be a humiliating experience: its cost to Hillary (regardless of his chance of winning the election) is 3 utils.

(b) Is there a separating pure-strategy equilibrium in which Hillary makes a different decision whether to go on Saturday Night Live, write a book or do nothing depending on whether she has a *high* or *low* chance of winning the election. If such equilibria exist, describe them and show they are equilibria. If no such equilibrium exists, explain why not.

(3) (*Hard*) **Nobility Doesn't Advertise**. Imagine a country where each person is either upper class, middle class or lower class, and there is an equal proportion of each type. Regardless of one's true class, being thought to be upper class is worth 120; being thought to be middle class is worth 90; and being thought to be lower class is worth 0. One way in which a person can try to indicate their class to others is by choosing 'to signal'. This signal is a crude 'yes-or-no' choice; that is, each person can only choose whether to signal or not. The cost of the signal is 200 for lower-class types; 10 for middle-class types; and 0 for upper-class types. Whether or not a person chooses to signal, after he or she has made that decision, he or she must take a compulsory 'high-society' test. The outcome of the test is completely independent of whether or

not a person has signaled. Upper-class types always pass the test; lower-class types never pass; and middle-class types pass with probability one half.

(a) Suppose there were an equilibrium in which only the middle class signaled. In such an equilibrium, what must society believe when it sees a person signaling and passing the test; signaling and failing; not signaling and passing; or not signaling and failing?

(b) Show carefully that there is indeed an equilibrium in which only middle-class types signal.

(c) [Hard] Show that there is also an equilibrium in which both middle- and upper-class types signal. To do this, you may find it helpful to assume society believes that anyone who does not signal and who passes the test is middle class.

(4) An Auction. Alice and Bob would both like to own the same hand-written and signed manuscript of a Frost poem. The manuscript is worth \$5 million to Alice and worth \$3 million to Bob. The present owner of the manuscript proposes the following method of sale, known as a 'second-price auction'. Alice and Bob will each simultaneously write down a 'bid' for the manuscript. Let b_A be Alice's bid, and let b_B be Bob's bid. The manuscript will go to the person whose bid is highest, and that person will have to pay **the other person**'s bid. If the bids are tied, then a fair coin will be tossed to decide who gets the manuscript, and that person will have to pay the tied bid. In either case, the person who does not get the manuscript pays nothing.

Here then is Alice's payoff function in this game (in \$ millions):

$$u_A(b_A, b_B) = \begin{cases} 5 - b_B & \text{if } b_A > b_B \\ 0 & \text{if } b_B > b_A \\ \frac{1}{2}(5 - b_B) & \text{if } b_A = b_B \end{cases}$$

Everything above is common knowledge to the players (in particular, they know each other's payoffs). There are no other bidders. Negative bids are not allowed. [*Notice that you do not need to answer parts (c) and (d) in order to answer part (e).*]

(a) Write down Bob's payoff function $u_B(b_A, b_B)$.

(b) What are Alice's best responses to $b_B = 2$? What are Alice's best responses to $b_B = 10$?.

(c) Are the following strategy profiles equilibria of the game: $(5, 3)$; $(4, 7)$; $(2, 3)$; $(10, 0)$ where the first number is b_A and the second is b_B. Explain your answer.

(d) Are there any equilibria in which Bob wins the manuscript? Explain.

(e) Which (if any) of Alice's and Bob's strategies are strictly dominated. Which (if any) are weakly dominated? Explain concisely but carefully.