Discussion of the Linear-City Differentiated Product Model

Ben Polak, Econ 159a/MGT522a

October 2, 2007

The Model.

- We can think a 'city' as a line of length one.
- There are two firms, 1 and 2, at either end of this line.
  - The firms simultaneously set prices P_1 and P_2 respectively.
  - Both firms have constant marginal costs.
  - Each firm's aim is to maximize its profit.
- Potential customers are evenly distributed along the line, one at each point.
  - Let the total population be one (or, if you prefer, think in terms of market shares).
- Each potential customer buys exactly one unit, buying it either from firm 1 or from firm 2.
  - A customer at position y on the line is assumed to buy from firm 1 if and only if

$$P_1 + ty^2 < P_2 + t(1-y)^2. \tag{1}$$

Interpretation. Customers care about both price and about the 'distance' they are from the firm. If we think of the line as representing geographical distance, then we can think of the $t \times (\text{distance})^2$ term as the 'transport cost' of getting to the firm. Alternatively, if we think of the line as representing some aspect of product quality — say, fat content in ice-cream — then this term is a measure of the inconvenience of having to move away from the customer's most desired point. As the transport-cost parameter t gets larger, we can think of products becoming more differentiated from the point of view of the customers. If $t = 0$ then the products are perfect substitutes.

What happens?

- The first thing to notice is that neither firm i will ever set its price $p_i < c$. Why?
- Second: if firm 2 sets price p_2, then firm 1 can capture the entire market if its sets its price just under $p_2 - t$. Why?
  - So, it is never a best response for firm 1 to set a price less than a penny under $p_2 - t$.
- But, can firm 1 do better by setting a price higher than $p_2 - t$?
  - The downside is that it will give up some of the market.
  - The upside is that it will charge more to any customers it keeps.
- To answer this, we need to figure out exactly what is firm 1's share of the market (and hence profit) at any price combination.

Demands and profits if the market is split. Suppose that prices P_1 and P_2 are close enough that the market is split between the two firms. How do we calculate how many customers buy from firm 1?

- Answer: find the position, x, of an indifferent customer.
  - all customers to her left ($< x$) will strictly prefer to buy from firm 1.
  - all customers to her right ($> x$) will strictly prefer to buy from firm 2.

To find x, use expression (1) and set $P_1 + tx^2 = P_2 + t(1-x)^2$. Solve for x to get firm 1's demand when prices are 'close':

$$D(P_1, P_2) = x = \frac{P_2 + t - P_1}{2t} \tag{2}$$

Now, we can use this demand function to calculate firm 1's profits. Provided prices are 'close', firm 1's profit is given by

$$\pi_1(P_1, P_2) = (P_1 - c)D(P_1, P_2) = (P_1 - c)\left(\frac{P_2 + t - P_1}{2t}\right) \tag{3}$$

Firm 1's Best Response. How do we find firm 1's best response to each P_2? At least when prices are close, we can see which price P_1 maximizes the profit function in expression (3). Using calculus (the product rule), we obtain the first order condition

$$\left(\frac{P_2 + t - P_1^*}{2t}\right) + (P_1^* - c)\left(\frac{-1}{2t}\right) = 0 \tag{4}$$

which simplifies to

$$P_1^* = \frac{P_2 + t + c}{2}. \tag{5}$$

(Notice in passing that this price is exactly half way between the competitive price c and the price at which firm 1 gets no demand at all $P_2 + t$. Similarly, if a monopolist faces a linear demand curve $p = a - bq$, and has constant marginal costs c, the monopoly price is $\frac{a+c}{2}$: half way between the no demand price a and the competitive price c).

Drawing the Best Response Function. See figure 1 on page 4.

1. First draw the line $P_1 = c$. We know that firm 1's best response function, $BR_1(P_2)$ never goes to the left of this line. Why?
2. Next, draw the line $P_1 = P_2 - t$. We know that $BR_1(P_2)$ never goes more than a penny to the left of this line. Why?
3. Next, we draw the line $P_1 = \frac{P_2+t+c}{2}$, from expression (5).
   - To help us draw this, notice that when $P_2 = c - t$, we get $P_1 = c$. Draw this point.

- Then notice that for each unit increase in P_2, we increase P_1 by half a unit. Draw this line.

A rough picture of the best response function is shown by the bold line in figure 1 on page 4. (This is rough (a) because at very low values of P_2, the best response of firm 1 is any price high enough to ensure no demand; and (b) because at very high values of P_2, the best response of firm 1 is to price just slightly to the left of the line $P_2 - t$ shown).

Finding the Nash Equilibrium. Since the model is symmetric, firm 2's best response is similar to firm 1's but reflected in the 45° line. Both best response functions are shown in figure 2 (page 4). We can see that the NE is where the lines cross.

- To solve explicitly, plug $P^* = P_1^* = P_2^*$, into expression (5), to get $P^* = \frac{P^*+t+c}{2}$, or

$$P^* = c + t.$$

Try redrawing the graph for different values of t, and confirm that the Nash equilibrium price moves as the algebra predicts.

Economic Implications. Recall that we liked the Bertrand model because it is plausible that firms compete in prices. But we disliked the conclusion of the Bertrand model: that two firms are enough to get competitive prices $P^B = c$. By introducing differentiated products, we have kept the plausible part of the model while also getting a plausible conclusion.

- The equilibrium mark up over costs is not zero, but t.
  - The larger are the 'transport costs' t of moving from product to product, the higher are the equilibrium prices (and hence profits).
  - If there are no such transport or taste costs (i.e., goods are homogeneous) once again prices equal marginal costs.
  - Firms like product differentiation (product niches).
- But, we are holding the number of firms fixed. Entry may change our story.

Game Theory Lessons.

1. One thing we learn here is that "a little realism can help". Removing the extreme assumption of perfect substitutes gave us a model that seems more plausible.
2. Our methods are quite powerful. This was a complicated enough model for it not to be immediately obvious what would happen. But, by simply going through the steps we learned in class (find the best responses; find where they 'cross' etc.), we were able to solve the model relatively easily.