Final Exam

Econ 159a   Ben Polak

Fall 2005

This is a closed-book exam. The exam lasts for **120** minutes (plus **60** minutes reading time). There are **120** total points available (plus 5 extra credit). The exam has **4** questions and **6** pages. **Please put each question into a separate blue book.**

Question 1. [**30 total points. Use blue book 1.**] State whether each of the following claims is true or false (or can not be determined). For each, explain your answer in (at most) **one** short paragraph. Each part is worth **5** points**, of which 4 points are for the explanation.** Explaining an example or a counter-example is sufficient. Absent this, a nice concise intuition is sufficient: you do not need to provide a formal proof. Points will be deducted for incorrect explanations.

(a) "In the penalty-shot game, you should not shoot toward the middle of the goal (unless you are playing against England or Portsmouth [Ben" team])".

(b) "The reason that players cannot achieve a good outcome in the prisoners' dilemma is that they cannot communicate".

(c) "In a second-price auction, with private values, bidding more than one's true value is a weakly dominated strategy".

(d) "In duel (the game with the sponges) if a player knows that (were she not to shoot now) her opponent would shoot next turn, and knows that her opponent will have less than a 20% chance of hitting, then she should just wait and let her opponent shoot."

(e) "Consider a mixed-strategy equilibrium in which player i puts (positive) weight both on her strategy a and on her strategy b. Suppose we change the game such that all the payoffs to strategy a are increased slightly, while leaving all the payoffs to strategy b unchanged. Since a and b were indifferent for player i before, there cannot be a mixed-strategy equilibrium in the new game in which player i puts (positive) weight on strategy a and on strategy b."

(f) "In the alternating-offer bargaining game, if there are exactly three stages (with player 1 making the offers in stages one and three, and player two making the offer in stage two), then the equilibrium share offered in the first stage by player 1 to player 2 (i.e., the share that player 2 would get if he were to accept the offer) is decreasing in player 1's discount factor δ_1 (holding δ_2 fixed).

USE BLUE BOOK 1

USE BLUE BOOK 2

Question 2. [**34 total points + extra credit part**] **"Quality Tea"**

Barry has a company that makes tea. His only customer is Andrew. Barry has to decide whether to make his tea good or bad. Good tea is more expensive to make. Andrew has to decide whether to buy one or two bottles. All the bottles in a given production run are of the same quality. Andrew can not tell the quality of the tea when he decides how much to buy, but he does discover the quality later once he drinks it.

Andrew's payoff is 3 if he buys two bottles of tea and it is good; 2 if he buys one bottle and it is good; 1 if he buys one bottle and it is bad; and 0 if he buys two bottles and it is bad. Barry's payoff is: 3 if he makes bad tea and sells two bottles; 2 if he makes good tea and sells two bottles; 1 if he makes bad tea and sells one bottle; and 0 if he makes good tea and sells one bottle.

Notice that you can do parts (d)-(f) without doing parts (b)-(c).

(a) [**6 points**] Write down payoff matrix for this game. Find the Nash equilibrium.

Now suppose that Andrew and Barry have an on-going business relationship. That is, in each period, Barry has to choose the tea quality for that period; Andrew has to choose the quantity to purchase that period; and payoffs are realized for that period (i.e., the tea is consumed). Let δ_A be Andrew's discount factor, and let δ_B be Barry's discount factor.

(b) [**6 points**] First consider the case where the game is played just twice and then ends (i.e., there are just two periods). And, to keep things simple, assume $\delta_A = \delta_B = 1$. Find the SPE of this game. Be careful to write down a complete strategy for each player, and to explain your answer.

(c) [**8 points**] Now consider the case where the game is infinitely repeated, where $0 \leq \delta_A < 1$ and $0 \leq \delta_B < 1$. Find an SPE of this game in which, along the equilibrium path (i.e., if no-one deviates), Barry makes good tea in each period and Andrew buys two bottles in each period. Be careful to write down a complete strategy for each player, and to explain why your proposed strategy profile is an SPE. If it depends on δ_A and δ_B, specify the minimum δ_A and minimum δ_B such that your strategy is an SPE.

Question 2 continues on the next page: please turn over.

USE BLUE BOOK 2

USE BLUE BOOK 2

Question 2 continued.

Now suppose that, instead of choosing the quality of his tea afresh in each period, Barry must set the quality once and for all before period one. That is, whatever quality Barry chooses for the first period is then fixed for the rest of the game. Andrew, as before, makes a fresh choice of one or two bottles each period. Andrew knows that Barry's tea quality is fixed, but initially (until he drinks it) he does not observe whether Barry has fixed it as good or fixed it as as bad. The payoffs in each period are the same as before except that, if Barry fixes his tea quality as good, then Barry's payoff if Andrew buys two bottles in a period is reduced from 2 to 1.9. Let $\delta_A = \delta_B = 1$.

(d) [**6 points**] Suppose there are just two periods. Write down the tree for this game being careful to indicate what Andrew knows and when he knows it.

(e) [**8 points**] Find the SPE of this game. Show your work or explain your answer.

(f) [**Extra credit: 5 points to count against errors only.**] What is the minimum number of periods in this game for there to be an SPE in which Barry makes good tea. [Hint: although you can, you do not have to redraw the tree.]

USE BLUE BOOK 2

USE BLUE BOOK 3

Question 3. [**36 total points**] "**Wars of Attrition**".

Consider the following variant of a war of attrition. In each period, first firm A decides whether to fight or quit. Then, *after observing firm A's choice*, firm B decides whether to fight or to quit. If both firms quit, both firms get 0 payoff that period and the game ends. If firm A fights and firm B quits then firm A gets $5 - c_A$ that period, firm B gets 0 that period, and the game ends. If firm B fights and firm A quits then firm B gets $5 - c_B$ that period, firm A gets 0 that period, and the game ends. If both firms fight then firm A gets a payoff of $-c_A$ that period, firm B gets a payoff of $-c_B$ that period, and the game continues to the next period. The game continues until at least one firm quits or until the end of period 5. If both firms fight in period 5 then firm A gets a payoff of $-c_A$ that period, firm B gets a payoff of $-c_B$ that period, and the game ends without anyone getting the prize of 5.

Notice there are three differences from the usual war of attrition we discussed in class. First, in each period reached, firm B observes firm A's move before deciding her move. Second, if a firm elects to fight in a given period then its per period cost of fighting is paid even if the other side quits that period. Third, those costs c_A and c_B (which are specified below) need not be symmetric.

For Nerds only: you can assume that there is no discounting (i.e., each player aims to maximize the sum of her payoffs over the course of the game); and that it is commonly known that everyone has taken this course.

(a) [**6 points**] Let $c_A = c_B = 2$. Suppose we have reached period 4 of the game; that is, both players have fought in the first three periods. Explain how would you expect the game to proceed from this point?

(b) [**6 points**] Let $c_A = 3$ and $c_B = 2$. Suppose we have reached period 4 of the game; that is, both players have fought in the first three periods. Explain how would you expect the game to proceed from this point?

(c) [**6 points**] Explain how you would expect the game to proceed from the start first for the case $c_A = c_B = 2$ and then for the case $c_A = 3$ and $c_B = 2$.

(d) [**9 points**] Suppose now that in odd-numbered periods, as before, firm A moves first and firm B observes A's move before making her choice; but now, in even-numbered periods, firm B moves first and A observes this move before making her choice. Explain how would you expect the game to proceed from the start first for the case $c_A = c_B = 2$; and then for the case $c_A = 3$ and $c_B = 2$.

(e) [**9 points**] Return to the version of the game where firm A moves first in each period, odd or even. But now let $c_A = 1$ and suppose that firm A has a 'budget constraint' of 4 so that she can fight for at most four periods. Explain how would you expect the game to proceed from the start first for the case $c_A = 1$ and $c_B = 2$; and then for the case $c_A = 1$ and $c_B = 3$.

USE BLUE BOOK 3

Question 4. [**20 total points.**] **"Grade Inflation".**

Consider the following game. Ben has to give each student a grade. There are three types t of student: type 0's, type 6's and type 12's. Each student knows her own type, but Ben does not know it. All that Ben knows entering the game is that there are equal proportions (1/3) of each type.

Each student and Ben play the following game. The student "announces" to Ben a type a, which must be one of 0, 6 or 12. This announcement may or may not be her true type. Ben hears this announcement and then assigns the student a grade b out of the following seventeen possible grades, $\{0, 1, 2, 3, \ldots, 15, 16\}$. Let t denote the type of the student. Let a be what the student "announces" her type to be. And let b be the grade she is assigned by Ben.

Payoffs for this game are as follows (read carefully). Ben would like to set the grade for each student equal to her type. Specifically, if Ben assigns grade b to a student of type t then Ben's payoff is:

$$u_B(b; t) = \begin{cases} b - t & \text{if } t \geq b \\ t - b & \text{if } b > t \end{cases}$$

Each student would like to have a grade that is higher than her type. But students do not like to get too inflated a grade: each students ideal grade is her type plus 4. Specifically, if Ben assigns grade b to a student of type t then the student's payoff is:

$$u_S(b; t) = \begin{cases} b - (t + 4) & \text{if } (t + 4) \geq b \\ (t + 4) - b & \text{if } b > (t + 4) \end{cases}$$

Notice that these payoffs do not depend *directly* on a, but announcements might affect payoffs indirectly: Ben's grade assignment b might depend on a.

(a) [**4 points**] Explain briefly why, regardless of the student's announcement, Ben will never assign a grade strictly greater than 12.

(b) [**5 points**] Consider the following strategy profile. Each student truthfully announces her type, 0, 6 or 12 (that is, each student sets $a = t$). Ben believes the announcement and assigns a grade equal to the announcement (that is, Ben sets $b = a$). Regardless of whether or not this strategy profile is an equilibrium, what payoffs would result for each type of student and what average payoffs would result for Ben? Notice that, given the choices of the students, Ben is assigning grades optimally. Explain carefully whether or not this is an equilibrium.

Question 4 continues on the next page: please turn over.

USE BLUE BOOK 4

Question 4 continued.

(c) [**5 points**] Consider the following strategy profile. All students of all types announce they are type 12. Ben identifies all students (regardless of their announcements) to be equally likely to be each type, and assigns them all grade 6. Regardless of whether or not this strategy profile is an equilibrium, what payoffs would result for each type of student and what average payoffs would result for Ben? Notice that, given the choices of the students, Ben is assigning grades optimally. Explain carefully whether or not this is an equilibrium.

(d) [**6 points**] Consider the following strategy profile. Students of type 0 announce they are type 6. Students of type 6 announce they are type 12. And students of type 12 also announce they are type 12. Ben identifies students who announce they are type 6 to be type 0, and he assigns them grade 0. Ben identifies students who announce they are type 12 to be equally likely to be type 6 or type 12, and he assigns them grade 9. (And, for completeness, if Ben were to see an announcement of 0 by a student, he would identify that student to be of type 0 and assign her grade 0). Regardless of whether or not this strategy profile is an equilibrium, what payoffs would result for each type of student and what average payoffs would result for Ben? Notice that, given the choices of the students, Ben is assigning grades optimally. Explain carefully whether or not this is an equilibrium.