Lecture 15 29 Oct 07

Zeimelo Theorem

- 2 players
- perfect information
- finite nodes
- three (or two) outcomes W, L, T

Either 1 can force a win (for 1)
or 1 can force a tie
or 2 can force a loss (on 1)

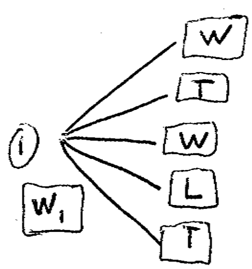
e.g. Nim unequal → 1 can force a W_1
equal → 2 can force a L_1

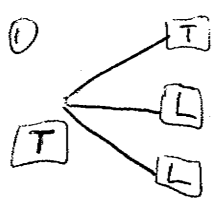
e.g. T.T.T. → tie

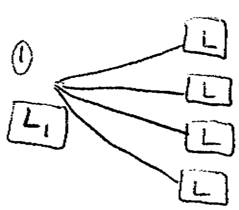
e.g. Chess

Proof (by induction) on maximum length of game N

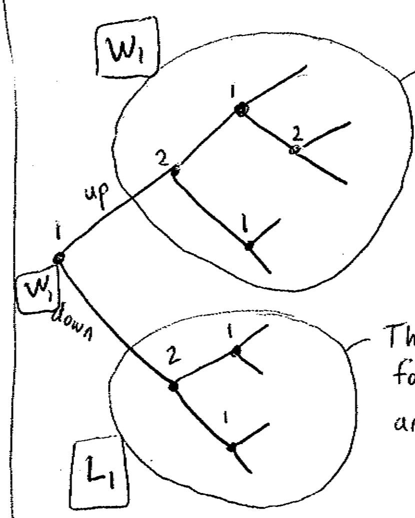
- if $N=1$







- Suppose the claim is true for all games of length $\leq N$
We claim therefore it will be true for games of length $N+1$



This is a subgame following 1 choosing up and it has length 3

This is a subgame following 1 choosing down and it has length 2

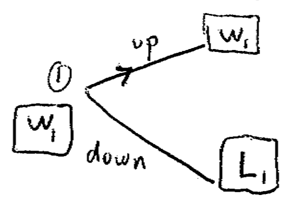
example $N=3$
$N+1=4$

Induction hypothesis

By induction hypothesis, upper subgame has a solution. Say, $\boxed{W_1}$.

By induction hypothesis, lower subgame has a solution. Say, $\boxed{L_1}$.

So translate the above game to:

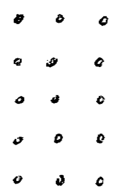


This has a solution, it is a game of length 1.

- Claim: we're done with proof (by induction).
  1 solution ✓
  2 initial step, then game of 1 (solution). solution ✓
  3 b/c 2 has solution, ⇒ 3 solution. ✓
  ⋮

Open Yale courses

example



array of nodes $N \times M$

Zeimelo's Thm → this game has a solution (which could depend on $N \times M$)

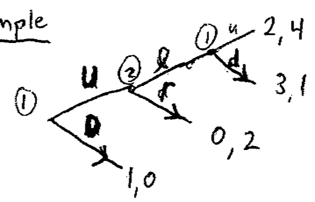
Homework $\boxed{\text{what is the solution}}$

Formal Stuff

Defn A game of perfect information is one in which at each node, the player whose turn it is to move knows which node she is at (and how she got there).

Defn A pure strategy for player i in a game of perfect information is a complete plan of actions; it specifies which action i will take at each of its decision nodes.

example



player 2 strategies [l] [r]

player 1 strategies [U,u] [U,d]

$\underbrace{[D,u] \quad [D,d]}_{\text{redundant}}$

BI : ([D,d], r)

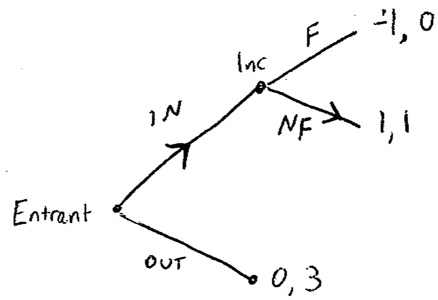
	l	r
Uu	2,4	0,2
Ud	3,1	0,2
Du	1,0	1,0
Dd	1,0	1,0

found by BI

equilibrium ??

NE = ([Dd], r), ([Du], r)

Danger – Finding outcomes that will never be reached



		Inc F	Inc NF
Ent	IN	-1,0	1,1
	OUT	0,3	0,3

NE = (IN, NF) ← BI

(OUT, F) ← ?? What is happening with this equilibrium ?

It is a NE but relies on believing an $\boxed{\text{incredible threat}}$