

# Partial Answers to the 2005 Final Exam

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PLEASE NOTE: THESE ARE ROUGH ANSWERS. I WROTE THEM QUICKLY SO I AM CAN'T PROMISE THEY ARE RIGHT! SOMETIMES I HAVE WRITTEN MORE THAN YOU NEED TO. SOMETIMES I HAVE WRITTEN LESS THAN WOULD BE WISE: JUST ENOUGH (I HOPE) TO GIVE AN IMPRESSION OF THE RIGHT ANSWER. I HAVE NOT DRAWN TREES SO THIS MAKES SOME OF THE ANSWERS BELOW RATHER CUMBERSOME.

Question 1. [30 total points. Use blue book 1.] State whether each of the following claims is true or false (or can not be determined). For each, explain your answer in (at most) **one** short paragraph. Each part is worth **5** points, **of which 4 points are for the explanation**. Explaining an example or a counter-example is sufficient. Absent this, a nice concise intuition is sufficient: you do not need to provide a formal proof. Points will be deducted for incorrect explanations.

- (a) “In the penalty-shot game, you should not shoot toward the middle of the goal (unless you are playing against England or Portsmouth [Ben” team])”.

**Answer:** *True (if we use the probabilities of scoring from class). If we think it more likely that the goalie will dive to the left, then we should shoot to the right. If we think it more likely that the goalie will dive to the right, then we should shoot to the left. Shooting to the middle is never a best response.*

- (b) “The reason that players cannot achieve a good outcome in the prisoners’ dilemma is that they cannot communicate”.

**Answer:** *False. Regardless of what the other player tells me, non-cooperation is a dominant strategy. To escape this outcome, we need something that changes the payoffs, like a contract (or repetition).*

- (c) “In a second-price auction, with private values, bidding more than one’s true value is a weakly dominated strategy”.

**Answer:** *True. Consider bidding one’s true value  $v_i$  instead of bidding something more, say  $v_i + 2$ . The only time these bids lead to different outcomes is if the highest bid of other players lies between  $v_i$  and  $v_i + 2$ , say  $v_i + 1$ . In this case, bidding  $v_i$  yields a payoff of 0, but bidding  $v_i + 2$  yields a payoff of  $[v_i - (v_i + 1)] = -1$ , which is worse. The exact choice of 2 and 1 here is irrelevant.*

- (d) “In duel (the game with the sponges) if a player knows that (were she not to shoot now) her opponent would shoot next turn, and knows that her opponent will have less than a 20% chance of hitting, then she should just wait and let her opponent shoot.”

**Answer:** *False. She should throw now if her probability of hitting now is greater than her opponent’s probability of missing tomorrow. We know that the opponent’s probability of missing tomorrow is greater than 80% (say, 85%), but this might not be as high as the player’s probability of hitting today (which could be 90% for example).*

- (e) “Consider a mixed-strategy equilibrium in which player  $i$  puts (positive) weight both on her strategy  $a$  and on her strategy  $b$ . Suppose we change the game such that all the payoffs to strategy  $a$  are increased slightly, while leaving all the payoffs to strategy  $b$  unchanged. Since  $a$  and  $b$  were indifferent for player  $i$  before, there cannot be a mixed-strategy equilibrium in the new game in which player  $i$  puts (positive) weight on strategy  $a$  and on strategy  $b$ .”

**Answer:** *False. Suppose there are two players and, in the original equilibrium, neither player were playing a pure strategy. In the new equilibrium, the mixed strategy of the other player will change to maintain indifference for player  $i$ , but player  $i$ ’s equilibrium mix will not change.*

- (f) “In the alternating-offer bargaining game, if there are exactly three stages (with player 1 making the offers in stages one and three, and player two making the offer in stage two), then the equilibrium share offered in the first stage by player 1 to player 2 (i.e., the share that player 2 would get if he were to accept the offer) is decreasing in player 1’s discount factor  $\delta_1$  (holding  $\delta_2$  fixed).

**Answer:** *True. As we raise  $\delta_1$ , player 2 has to offer more to player 1 in stage two to get player 1 to accept (in fact, the least she will accept is  $\delta_1$ ). This makes reaching stage two of the game less valuable to player 2, so player 1 has to offer less to player 2 in stage one to get player 2 to accept.*

## USE BLUE BOOK 1

**USE BLUE BOOK 2**

**Question 2. [34 total points + extra credit part] “Quality Tea”**

Barry has a company that makes tea. His only customer is Andrew. Barry has to decide whether to make his tea good or bad. Good tea is more expensive to make. Andrew has to decide whether to buy one or two bottles. All the bottles in a given production run are of the same quality. Andrew can not tell the quality of the tea when he decides how much to buy, but he does discover the quality later once he drinks it.

Andrew’s payoff is 3 if he buys two bottles of tea and it is good; 2 if he buys one bottle and it is good; 1 if he buys one bottle and it is bad; and 0 if he buys two bottles and it is bad. Barry’s payoff is: 3 if he makes bad tea and sells two bottles; 2 if he makes good tea and sells two bottles; 1 if he makes bad tea and sells one bottle; and 0 if he makes good tea and sells one bottle.

Notice that you can do parts (d)-(f) without doing parts (b)-(c).

- (a) [6 points] Write down payoff matrix for this game. Find the Nash equilibrium.

**Answer:** *The NE, (one,bad), is indicated by the underlining.*

		<i>B</i>	
		bad	good
<i>A</i>	one	<u>1</u> , <u>1</u>	2, 0
	two	0, <u>3</u>	<u>3</u> , 2

Now suppose that Andrew and Barry have an on-going business relationship. That is, in each period, Barry has to choose the tea quality for that period; Andrew has to choose the quantity to purchase that period; and payoffs are realized for that period (i.e., the tea is consumed). Let  $\delta_A$  be Andrew’s discount factor, and let  $\delta_B$  be Barry’s discount factor.

- (b) [6 points] First consider the case where the game is played just twice and then ends (i.e., there are just two periods). And, to keep things simple, assume  $\delta_A = \delta_B = 1$ . Find the SPE of this game. Be careful to write down a complete strategy for each player, and to explain your answer.

**Answer:** *We know that, regardless of what has happened in the first stage, the players play the NE, (one,bad), in the second stage of the game. Therefore, in the first stage, Barry has no incentive to make good tea. Hence, in the first stage, they also play (one,bad). Formally, Andrew’s strategy is “one” followed by “one” regardless of what happens in stage one. And Barry’s strategy is “bad” followed by “bad” regardless of what happens in stage one.*

- (c) [8 points] Now consider the case where the game is infinitely repeated, where  $0 \leq \delta_A < 1$  and  $0 \leq \delta_B < 1$ . Find an SPE of this game in which, along the equilibrium path (i.e., if no-one deviates), Barry makes good tea in each period and Andrew buys two bottles in each period. Be careful to write down a complete strategy for each player, and to explain why your proposed strategy profile is an SPE. If it depends on  $\delta_A$  and  $\delta_B$ , specify the minimum  $\delta_A$  and minimum  $\delta_B$  such that your strategy is an SPE.

**Answer:** Consider the strategy profile in which Barry makes good tea as long as Barry has always made good tea and Andrew has always bought two bottles; but Barry makes bad tea for ever if Barry has ever made bad tea or Andrew has ever bought one bottle. Andrew buys two bottles as long as Barry has always made good tea and Andrew has always bought two bottles; but Andrew buys one bottle for ever if Barry has ever made bad tea or Andrew has ever bought one bottle. As long as Barry is playing this strategy, Andrew has no incentive to deviate: his choice is a best response in each stage game even if it were a one-shot game. Similarly, Barry has no incentive to deviate from 'bad' once Andrew is buying one bottle for ever. But we need Barry to have an incentive to make good tea as long as Andrew is buying two bottles. Our usual incentive equation becomes

$$[3 - 2] \stackrel{?}{\leq} \delta_B \left[ \frac{2}{1 - \delta_B} - \frac{1}{1 - \delta_B} \right]$$

which simplifies to  $\delta_B \geq 1/2$ . Notice that there is no minimum value of  $\delta_A$ .

**Question 2 continues on the next page: please turn over.**

**USE BLUE BOOK 2**

USE BLUE BOOK 2

**Question 2 continued.**

Now suppose that, instead of choosing the quality of his tea afresh in each period, Barry must set the quality once and for all before period one. That is, whatever quality Barry chooses for the first period is then fixed for the rest of the game. Andrew, as before, makes a fresh choice of one or two bottles each period. Andrew knows that Barry's tea quality is fixed, but initially (until he drinks it) he does not observe whether Barry has fixed it as good or fixed it as as bad. The payoffs in each period are the same as before except that, if Barry fixes his tea quality as good, then Barry's payoff if Andrew buys two bottles in a period is reduced from 2 to 1.9. Let  $\delta_A = \delta_B = 1$ .

- (d) [6 points] Suppose there are just two periods. Write down the tree for this game being careful to indicate what Andrew knows and when he knows it.

**Answer:** *To save my drawing the tree, I am going to provide you with a description of it. The simplest tree has Barry moving first choosing good or bad. The two nodes at the end of these edges form one information set for Andrew who (at each) chooses one or two. Each of the four nodes at the end of these edges are singleton information sets for Andrew, who again chooses one or two. The payoffs at the eight end nodes are given in the following table with Andrew's payoff given first.*

b11	b12	b21	b22	g11	g12	g21	g22
2, 2	1, 4	1, 4	0, 6	4, 0	5, 1.9	5, 1.9	6, 3.8

- (e) [8 points] Find the SPE of this game. Show your work or explain your answer.

**Answer:** *The easiest thing is to mark the best responses on the tree but absent the tree: in the second stage, Andrew chooses (one if bad) and (two if good) regardless of Andrew's first choice. Thus, if Andrew chooses one in the first stage then Barry's payoffs are 2 if he chooses bad and 1.9 if he chooses good. And if Andrew chooses two in the first stage then Barry's payoffs are 4 if he chooses bad and 3.8 if he chooses good. That is, bad strictly dominates good for Barry. Hence he will choose bad and Andrew will choose one in the first stage.*

- (f) [Extra credit: 5 points to count against errors only.] What is the minimum number of periods in this game for there to be an SPE in which Barry makes good tea. [Hint: although you can, you do not have to redraw the tree.]

**Answer:** *With three periods, if Barry chooses good and Andrew chooses two in the first stage (and two in the next two stages when he learns it is good), then Barry's payoff is  $(3 \times 1.9) = 5.7$ . If Barry deviates to bad in the first stage, then Barry gets 3 in the first stage plus 1 more each (once he is discovered) in stages two and three for a total of 5. Formally, Andrew's strategy here is two in the first stage and then (two,two) in the next two stages if the tea is good (regardless of what Andrew did in stage one), and (one,one) in the next two stages if the tea is bad (regardless of what Andrew did in stage one).*

USE BLUE BOOK 2

### USE BLUE BOOK 3

#### Question 3. [36 total points] “Wars of Attrition”.

Consider the following variant of a war of attrition. In each period, first firm  $A$  decides whether to fight or quit. Then, *after observing firm  $A$ 's choice*, firm  $B$  decides whether to fight or to quit. If both firms quit, both firms get 0 payoff that period and the game ends. If firm  $A$  fights and firm  $B$  quits then firm  $A$  gets  $5 - c_A$  that period, firm  $B$  gets 0 that period, and the game ends. If firm  $B$  fights and firm  $A$  quits then firm  $B$  gets  $5 - c_B$  that period, firm  $A$  gets 0 that period, and the game ends. If both firms fight then firm  $A$  gets a payoff of  $-c_A$  that period, firm  $B$  gets a payoff of  $-c_B$  that period, and the game continues to the next period. The game continues until at least one firm quits or until the end of period 5. If both firms fight in period 5 then firm  $A$  gets a payoff of  $-c_A$  that period, firm  $B$  gets a payoff of  $-c_B$  that period, and the game ends without anyone getting the prize of 5.

Notice there are three differences from the usual war of attrition we discussed in class. First, in each period reached, firm  $B$  observes firm  $A$ 's move before deciding her move. Second, if a firm elects to fight in a given period then its per period cost of fighting is paid even if the other side quits that period. Third, those costs  $c_A$  and  $c_B$  (which are specified below) need not be symmetric.

For Nerds only: you can assume that there is no discounting (i.e., each player aims to maximize the sum of her payoffs over the course of the game); and that it is commonly known that everyone has taken this course.

- (a) [6 points] Let  $c_A = c_B = 2$ . Suppose we have reached period 4 of the game; that is, both players have fought in the first three periods. Explain how would you expect the game to proceed from this point?

**Answer:** *Since costs are sunk, if we reach stage 5, then  $A$  will fight and  $B$  will quit. Hence, in stage four,  $A$  will fight and  $B$  will quit.*

- (b) [6 points] Let  $c_A = 3$  and  $c_B = 2$ . Suppose we have reached period 4 of the game; that is, both players have fought in the first three periods. Explain how would you expect the game to proceed from this point?

**Answer:** *It is the same: since costs are sunk, if we reach stage 5, then  $A$  will fight and  $B$  will quit. Hence, in stage four,  $A$  will fight and  $B$  will quit.*

- (c) [6 points] Explain how you would expect the game to proceed from the start first for the case  $c_A = c_B = 2$  and then for the case  $c_A = 3$  and  $c_B = 2$ .

**Answer:** *In either case, since we know that  $B$  will quit in stage four,  $A$  will fight and  $B$  will quit in stage three. And this argument repeats all the way to the first stage.*

- (d) [9 points] Suppose now that in odd-numbered periods, as before, firm  $A$  moves first and firm  $B$  observes  $A$ 's move before making her choice; but now, in even-numbered periods, firm  $B$  moves first and  $A$  observes this move before making her choice. Explain how would you expect the game to proceed from the start first for the case  $c_A = c_B = 2$ ; and then for the case  $c_A = 3$  and  $c_B = 2$ .

**Answer:** Let's do the  $c_A = c_B = 2$  case first. The analysis of stage five is the same. In stage four, even if  $B$  fights first,  $A$  knows that by fighting twice it will gain  $5 - (2 \times 2) = 1$  so it will fight, so  $B$  will quit. From here, we roll back as before: so in stage one,  $A$  fights and  $B$  quits. Now the case  $c_A = 3$  and  $c_B = 2$ . The analysis of stage five is the same. But now, in stage four, if  $B$  fights first,  $A$  knows that by fighting twice it will gain  $5 - (3 \times 3) = -1$  so  $A$  will quit, so  $B$  will fight. In stage three, even if  $A$  fights first,  $B$  knows that by fighting twice it will gain  $5 - (2 \times 2) = 1$  so it will fight, so  $A$  will quit. And from here, we roll back to the first stage in which  $A$  quits and  $B$  fights.

- (e) [9 points] Return to the version of the game where firm  $A$  moves first in each period, odd or even. But now let  $c_A = 1$  and suppose that firm  $A$  has a 'budget constraint' of 4 so that she can fight for at most four periods. Explain how would you expect the game to proceed from the start first for the case  $c_A = 1$  and  $c_B = 2$ ; and then for the case  $c_A = 1$  and  $c_B = 3$ .

**Answer:** Let's do the case  $c_A = 1$  and  $c_B = 2$  first. In stage five,  $A$  must quit so  $B$  will fight. In stage four, even if  $A$  fights first,  $B$  knows that by fighting twice it will gain  $5 - (2 \times 2) = 1$  so it will fight, so  $A$  will quit. And from here, we roll back to the first stage in which  $A$  quits and  $B$  fights. Now the case,  $c_A = 1$  and  $c_B = 3$ .

The analysis of stage five is the same. But now, in stage four, if  $A$  fights first,  $B$  knows that by fighting twice it will gain  $5 - (3 \times 3) = -1$  so  $B$  will quit, so  $A$  will fight. And from here, we roll back to the first stage in which  $A$  fights and  $B$  quits.

### USE BLUE BOOK 3

## USE BLUE BOOK 4

### Question 4. [20 total points.] “Grade Inflation”.

Consider the following game. Ben has to give each student a grade. There are three types  $t$  of student: type 0's, type 6's and type 12's. Each student knows her own type, but Ben does not know it. All that Ben knows entering the game is that there are equal proportions ( $1/3$ ) of each type.

Each student and Ben play the following game. The student “announces” to Ben a type  $a$ , which must be one of 0, 6 or 12. This announcement may or may not be her true type. Ben hears this announcement and then assigns the student a grade  $b$  out of the following seventeen possible grades,  $\{0, 1, 2, 3, \dots, 15, 16\}$ . Let  $t$  denote the type of the student. Let  $a$  be what the student “announces” her type to be. And let  $b$  be the grade she is assigned by Ben.

Payoffs for this game are as follows (read carefully). Ben would like to set the grade for each student equal to her type. Specifically, if Ben assigns grade  $b$  to a student of type  $t$  then Ben's payoff is:

$$u_B(b; t) = \begin{cases} b - t & \text{if } t \geq b \\ t - b & \text{if } b > t \end{cases}$$

Each student would like to have a grade that is higher than her type. But students do not like to get too inflated a grade: each student's ideal grade is her type plus 4. Specifically, if Ben assigns grade  $b$  to a student of type  $t$  then the student's payoff is:

$$u_S(b; t) = \begin{cases} b - (t + 4) & \text{if } (t + 4) \geq b \\ (t + 4) - b & \text{if } b > (t + 4) \end{cases}$$

Notice that these payoffs do not depend *directly* on  $a$ , but announcements might affect payoffs indirectly: Ben's grade assignment  $b$  might depend on  $a$ .

- (a) [4 points] Explain briefly why, regardless of the student's announcement, Ben will never assign a grade strictly greater than 12.

**Answer:** Ben is trying to get as close to the “correct” grade as possible. Since no student has a type higher than 12, the strategies  $\{13, \dots, 16\}$  are ‘dominated’ by 12. That is, the payoff to Ben from 12 is always higher than the payoff from these higher grades.

- (b) [5 points] Consider the following strategy profile. Each student truthfully announces her type, 0, 6 or 12 (that is, each student sets  $a = t$ ). Ben believes the announcement and assigns a grade equal to the announcement (that is, Ben sets  $b = a$ ). Regardless of whether or not this strategy profile is an equilibrium, what payoffs would result for each type of student and what average payoffs would result for Ben? Notice that, given the choices of the students, Ben is assigning grades optimally. Explain carefully whether or not this is an equilibrium.

**Answer:** For each student type,  $b = a = t$ , hence their payoff would be  $-4$ . Ben would get the grades exactly right so his payoff would be 0. This is not an equilibrium. For example, a type 0 student could announce 6 in which case Ben would set  $b = 6$  and the student would get a payoff of  $-2$ .

Question 4 continues on the next page: please turn over.

USE BLUE BOOK 4

## USE BLUE BOOK 4

### Question 4 continued.

- (c) [5 points] Consider the following strategy profile. All students of all types announce they are type 12. Ben identifies all students (regardless of their announcements) to be equally likely to be each type, and assigns them all grade 6. Regardless of whether or not this strategy profile is an equilibrium, what payoffs would result for each type of student and what average payoffs would result for Ben? Notice that, given the choices of the students, Ben is assigning grades optimally. Explain carefully whether or not this is an equilibrium.

**Answer:** Now students of type 0 get a payoff of  $(4 - 6) = -2$ ; students of type 6 get a payoff of  $(6 - 10) = -4$ , and students of type 12 get a payoff of  $(6 - 16) = -10$ . Ben gets a payoff of  $\frac{1}{3}(-6) + \frac{1}{3}(0) + \frac{1}{3}(-6) = -4$ . This is an equilibrium. Ben cannot do any better: given the absence of information revealed by the student announcement, he is making the best guess he can. The students can not do better since Ben is not responsive to their announcements.

- (d) [6 points] Consider the following strategy profile. Students of type 0 announce they are type 6. Students of type 6 announce they are type 12. And students of type 12 also announce they are type 12. Ben identifies students who announce they are type 6 to be type 0, and he assigns them grade 0. Ben identifies students who announce they are type 12 to be equally likely to be type 6 or type 12, and he assigns them grade 9. (And, for completeness, if Ben were to see an announcement of 0 by a student, he would identify that student to be of type 0 and assign her grade 0). Regardless of whether or not this strategy profile is an equilibrium, what payoffs would result for each type of student and what average payoffs would result for Ben? Notice that, given the choices of the students, Ben is assigning grades optimally. Explain carefully whether or not this is an equilibrium.

**Answer:** Now students of the type 0 get a payoff of  $(0 - 4) = -4$ . Students of type 6 get a payoff of  $(10 - 9) = 1$ . And students of type 12 get a payoff of  $(16 - 9) = 7$ . Ben gets a payoff of  $\frac{1}{3}(0) + \frac{1}{3}(-3) + \frac{1}{3}(7) = 0$ . The question does not specify what grade Ben will assign for other weird announcements such as 7 but let's assume Ben assigns all these grades of 0. Since these weird announcements never happen in the supposed equilibrium, it costs Ben nothing to make weird grade assignments to them. Given this rule, this is indeed an equilibrium. Ben cannot do any better: he distinguishes the 0-types but, given he cannot distinguish the 6-types from the 12-types, he is making the best guess he can. For type 0's, the only deviation that makes any difference is to announce 12 but then their payoff would fall to  $(4 - 9) = -5$ . For the type 6's and type 12's, given Ben's strategy, all deviation would be interpreted as if they were type 0's. This would reduce their payoffs to  $-10$  and  $-16$  respectively.