The Atmosphere, Ocean and Environmental Change — Due Friday
EVST201a/G&G 140a — In class, Sept. 9
Problem Set #1

Although 140a/201a is partly a descriptive course, explanations based on geometry, algebra, thermodynamics and elementary physics are used frequently. In fact, the course satisfies QR credit in Yale College. This problem set will help you to review some quantitative subjects before getting deeply involved in the course. As you solve these problems, be careful to use and express the correct units for each physical quantity. By default, we use the S.I. system in this course (i.e. kilograms, meters, seconds, etc.) It is also recommended that for each problem, you make a sketch of the physical situation or the mathematical function involved. If you are uncertain about any of these problems, contact one of the Teaching Assistants or your College Science Tutor.

1. A cloud droplet 2 km above the earth is falling at a constant speed* of 3cm/sec. At that rate, how long will it take to reach the ground? Express your answer three ways: in seconds, in minutes, in hours. (*Droplets fall at constant speed after they accelerate to "terminal velocity"; the speed where the forces of gravity and air drag balance each other.)

2. A balloon 1500 meters above the earth's surface is seen by an observer at an elevation angle of 15°. What is the horizontal distance from the observer to the point on the ground directly below the balloon?

3. If the earth were perfectly spherical with a radius of 6370 km, what would be its volume? Express your answer in exponential notation (i.e. powers of ten).

4. Calculate the volume of a fluid layer (like the oceans or the atmosphere) covering the earth. Assume that the earth is a perfect sphere and has a radius of 6370 km and that the layer is 5 km thick. Do the problem two ways:
   a) Compute the volume of two spheres with radii 6370 km and 6375 km; then take the difference.

   b) Multiply the surface area of the earth times the layer thickness.

5. a) What is the circumference of the earth if its radius is 6370 km?

   b) If there are 360° of longitude around the earth, how many kilometers are in each degree at the equator?

   c) If an airplane is traveling eastward at 100 m/s along the equator, how long will it take to circumnavigate the globe? Express your answer in hours.

6. A river is 3 meters deep, 50 meters wide and the average flow speed is one meter per second, how many cubic meters of water flow past a point every second? How much mass of water flows by a fixed point every second?

7. *According to Newton's law an object will move at constant velocity, (or remain stationary) unless acted on by a force. If a force (F) is applied, the object will accelerate according to $F = m\,a$ where "m" is the mass of the object and "a" is its acceleration.*
   a. Consider a stationary object with a mass of 5 kg, acted on by a force of 10 Nt (Nt = kg m/s^2). What is its acceleration?

   b. If that acceleration continues, how long would it take for the object to moving at a speed of 40 m/s?

The next few questions involve the so called "exponential function".

$$f(t) = A\,e^{\pm t/\tau}$$

$$or\;\; f(x) = A\,e^{\pm x/L}$$

where A is called the "pre-exponential factor" and τ or L is the "characteristic time" or "characteristic length" of the problem. The exponential function describes any process where the rate of change of a quantity is proportional to the quantity itself. Examples include.

- *population growth with time*
- *radioactive decay with time*
- *cooling of a hot object with time*
- *decay of a light beam with distance into an absorbing medium*
- *decrease of atmospheric pressure with altitude*

Using a calculator with an exponential function, answer the following questions.

8. Assume that the population of the Earth is approximately described by the formula

$$P = P_0\,e^{t/\tau}$$

where P_0 is the population in the year 2000 (P_0 = 6,127,000,000), t is the time in years before (negative) or after (positive) the year 2000, and τ is the characteristic time for population growth ($\tau = 86$ years). What was the population in the year 1900?

What will the population be in the years 2100 and 2200? (Note the assumption of constant "proportional" growth rate.)

9. If an object is heated and then exposed to the air, it will lose heat at a rate which is proportional to the instantaneous difference in temperature between the object and the air. The decay of the object temperature back to the air temperature is given by

$$T = T_0 \, e^{-t/\tau}$$

Where T_0 is the initial temperature elevation of the object and T is the elevation of the object temperature at later times. The symbol "t" is time after the heated object was exposed to the air and "τ" is the characteristic time for the cooling. The characteristic time for cooling would depend on the mass of the object the amount of surface area and on the wind speed.

If a branding iron is heated to 2000°C above the air temperature (20°C), and if $\tau = 5$ minutes, what will its temperature be 5, 10, and 15 minutes after it begins to cool?

10. To a good approximation, the air pressure in the atmosphere decreases with distance above the ground according to

$$p = P_0 \, e^{-Z/L}$$

where "P" is the pressure at any altitude "Z", P_o is the pressure at the surface of the earth ($P_0 = 101300$ Pascals), and L is the "scale height" of the earth's atmosphere ($L = 8400$ meters). The scale height is a measure of the depth of the atmosphere. What is the pressure at altitudes of 1 meter, 10 meters, 100 meters, 1000 meters, 10,000 meters and 100,000 meters.

The next few questions concern issues of power, heat, heat capacity, and temperature. The following equation describes the change of temperature of an object when it loses or gains heat.

$$Q = M \bullet C \bullet \Delta T$$

where

Q = *added or lost heat (units: Joules)*

M = mass of the object (units: kg)

C = specific heat capacity i.e. heat capacity per unit mass (units: Joule/kg degree)

ΔT = change in temperature (units degrees C)

11. A cubic meter of water has a mass of 1000 kg (i.e. about as much as a car). Water has a specific heat capacity of 4218 J • deg^{-1} • kg^{-1}.

    If one cubic meter of water receives 10 million Joules of heat, how much will its temperature change?

12. The rate of heat (or energy) loss or gain per unit time is called "power" with units

    Watts = Joules per second

    How long would it take for a single 100 watt light bulb to use 10 million Joules of energy? Express your answer in seconds; and in hours.

13. The Power Company requires 11 gallons of fuel to generate the electricity to light a 75 watt light bulb for one year. How many Joules of electrical energy are generated from one gallon of fuel?

14. Consider a small pond with horizontal dimensions 100 meters by 100 meters, receiving heat from the sun at a rate of 500 watts per square meter.
    a. Over a 10 hour period, how much heat is received by the pond?

    b. If the depth of the pond is 2 meters, how many cubic meters of water are in the pond?
    c. Using information from questions 11 and 14 a, b, determine how much the temperature of the pond will rise over the 10 hour period of solar heating.

All homework should be kept in a notebook after it is returned to you. This file will be extremely valuable when studying for exams.