Midterm Exam

Econ 159a/MGT522a

Ben Polak Fall 2007

- This is a **closed-book** exam.
- There are **4** pages including this one.
- The exam lasts for **75** minutes.
- There are **75** total points available.
- There are **three** questions, worth **15, 30 and 30** points respectively.
- Please put **each** question into a **different blue book.**
- Show your work.

Question 1. [**15 total points: use blue book 1**]. **Short-Answer Questions**.

State whether each of the following claims is true or false (or can not be determined). For each, explain your answer in (at most) **one short** paragraph**.** Each part is worth **5** points**, of which 4 points are for the explanation.** Explaining an example or a counter-example is sufficient. Absent this, a nice **concise** intuition is sufficient: you do not need to provide a formal proof. Points will be deducted for incorrect explanations.

(a) [**5 points**] "A strictly dominated strategy can never be a best response."

(b) [**5 points**] "In the candidate-voter model, if two people are standing, one to the left of center and one to the right of center, and neither of them is 'too extreme', then it is an equilibrium."

(c) [**5 points**] "If $(\hat{s}, \hat{s})$ is a Nash equilibrium of a symmetric, two-player game then $\hat{s}$ is evolutionarily stable."

Question 2. [30 total points] "Party Games". Roger has invited Caleb to his party. Roger must choose whether or not to hire a clown. Simultaneously, Caleb must decide whether or not to go the party. Caleb likes Roger but he hates clowns – he even hates other people seeing clowns! Caleb's payoff from going to the party is 4 if there is no clown, but 0 if there is a clown there. Caleb's payoff from not going to the party is 3 if there is no clown at the party, but 1 if there is a clown at the party. Roger likes clowns – he especially likes Caleb's reaction to them — but does not like paying for them. Roger's payoff if Caleb comes to the party is 4 if there is no clown, but $8 - x$ if there is a clown (x is the cost of a clown). Roger's payoff if Caleb does not come to the party is 2 if there is no clown, but $3 - x$ if there is a clown there.

(a) [6 points] Write down the payoff matrix of this game.

(b) [6 points] Suppose $x = 0$. Identify any dominated strategies. Explain. Find the Nash equilibrium. What are the equilibrium payoffs?

(c) [6 points] Suppose $x = 2$. Identify any dominated strategies. Explain. Find the Nash equilibrium. What are the equilibrium payoffs?

(d) [6 points] Suppose $x = 3$. Identify any dominated strategies. Explain. Find the Nash equilibrium. What are the equilibrium payoffs?

(e) [6 points] Suppose $x = 5$. Identify any dominated strategies. Explain. Find the Nash equilibrium. What are the equilibrium payoffs?

Question 3. [30 total points] "Road Trip". Six Yale students are going on a foreign trip on which they will live close together. Where they are going, there is a disease which spreads easily among people who live close together. The value of the trip to a student who does not get the disease is 6. The value of the trip to a student who gets the disease is 0.

There is a vaccination against the disease. The vaccination costs different amounts for different students (perhaps they have different health plans). Let's call the students $1, 2, 3, 4, 5$ and 6 respectively. The vaccination costs 1 for student 1; it costs 2 for student 2; etc....

If a student gets vaccinated, she will not get the disease. But, if she is not vaccinated then her probability of getting the disease depends on the total number in the group who are not vaccinated. If she is the only person not to get vaccinated then the probability that she gets the disease is 1/6. If there is one other person who is not vaccinated (i.e., two in all including her) then the probability that she gets the disease is 2/6. If there are two other people who are not vaccinated (i.e., three including her) then the probability that she gets the disease is 3/6, etc..

[For example, suppose only students 2 and 4 get vaccinated. Then 2's expected payoff is $6 - [2]$ where the $[2]$ is the cost of the vaccination. Student 4's expected payoff in this case is $6 - [4]$. Student 5's expected payoff in this case (recall she did not get vaccinated) is $\left(\frac{2}{6}\right) 6 + \left(\frac{4}{6}\right) 0 = 2$ where the fraction $\left(\frac{4}{6}\right)$ is the probability that she gets the disease.]

To make this into a game, suppose that each student aims to maximize her expected payoff. The students decide, individually and simultaneously, whether or not to get a vaccination.

(a) [8 points] Explain concisely whether or not it is a Nash equilibrium for students 1,2,3 and 4 to get vaccinated and students 5 and 6 not to get vaccinated.

(b) [8 points] Explain concisely whether or not it is a Nash equilibrium for students 1,2 and 3 to get vaccinated and students 4,5, and 6 not to get vaccinated.

(c) [6 points] Which players in this game have strictly or weakly dominated strategies? Explain your answers carefully including whether any domination is strict or weak.

(d) [4 points] If we delete all strictly and weakly dominated strategies from all players, which players in the game now have (iteratively) strictly or weakly dominated strategies? Explain carefully.

(e) [4 points] [Harder] Find *all* the (possibly mixed) NE in this game. Explain.