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Econ 252 - Financial Markets

Spring 2011

Professor Robert Shiller

Problem Set 6 – Solution

Question 1

(a) A futures contract is an obligation to trade at a future date at a price specified in the contract, and an options contract is a right, but not an obligation, to trade at a future date at a price specified in the contract. Investors use futures to protect against symmetric risk and options to protect against asymmetric risk.

(b) The correct answer is the second alternative. The farmer should sell futures contracts for 20,000 bushels with a futures price of $7 and delivery in 3 months. He then can sell at $7, no matter what the market price is. Therefore, he can make exactly $140,000 and pay back his loan.

If he purchases those futures contracts, then the futures contract obligates him to purchase corn in 3 months, which he does not need. If he buys put options with a strike price of $2, he obtains the right to sell the corn at $2 per bushel, and this would not be enough to cover his debt.

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(c) The correct answer is the third alternative. Because the manager believes that the stock price will down to \$25 in 6 months, he would like to place a bet reflecting this pessimistic outlook. So, the manager should purchase put options with a strike price of \$30 maturity maturing in 6 months.

If the investor purchases the call option from the first alternative, and the stock price falls below \$30, the option will be out of the money, and he might end up with a negative profit. If he purchases the future from the second alternative, then he gains if the spot price is higher than \$30 six months from now, but he incurs a loss if the stock price falls below \$30. As he believes that stock price will go down to \$25 in 6 months, neither this call nor this future will be appropriate for him.

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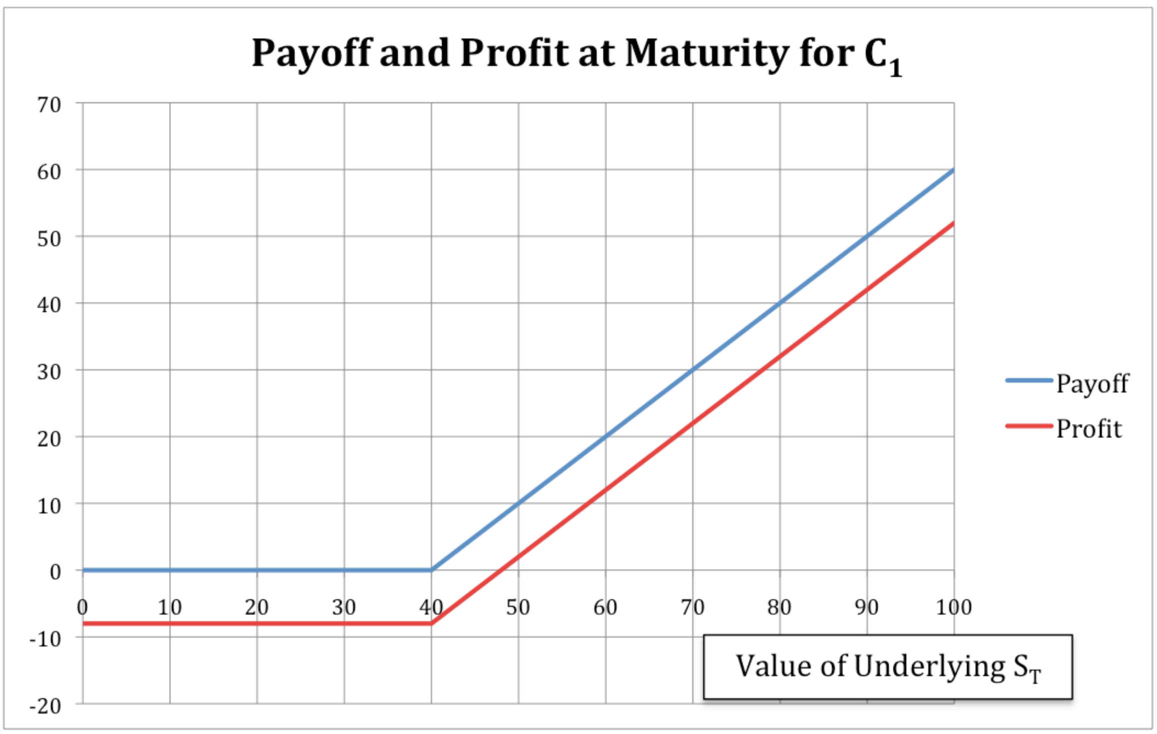
Question 2

(a) The payoff of the described call option at maturity is as follows:

Underlying	$S_T \leq 40$	$S_T > 40$
Payoff Call C_1 with E=40	0	S_T-40

The cost of the call option is $8. It follows that the profit of the described call option at maturity is as follows:

Underlying	$S_T \leq 40$	$S_T > 40$
Profit Call C_1 with E=40	-8	S_T-48



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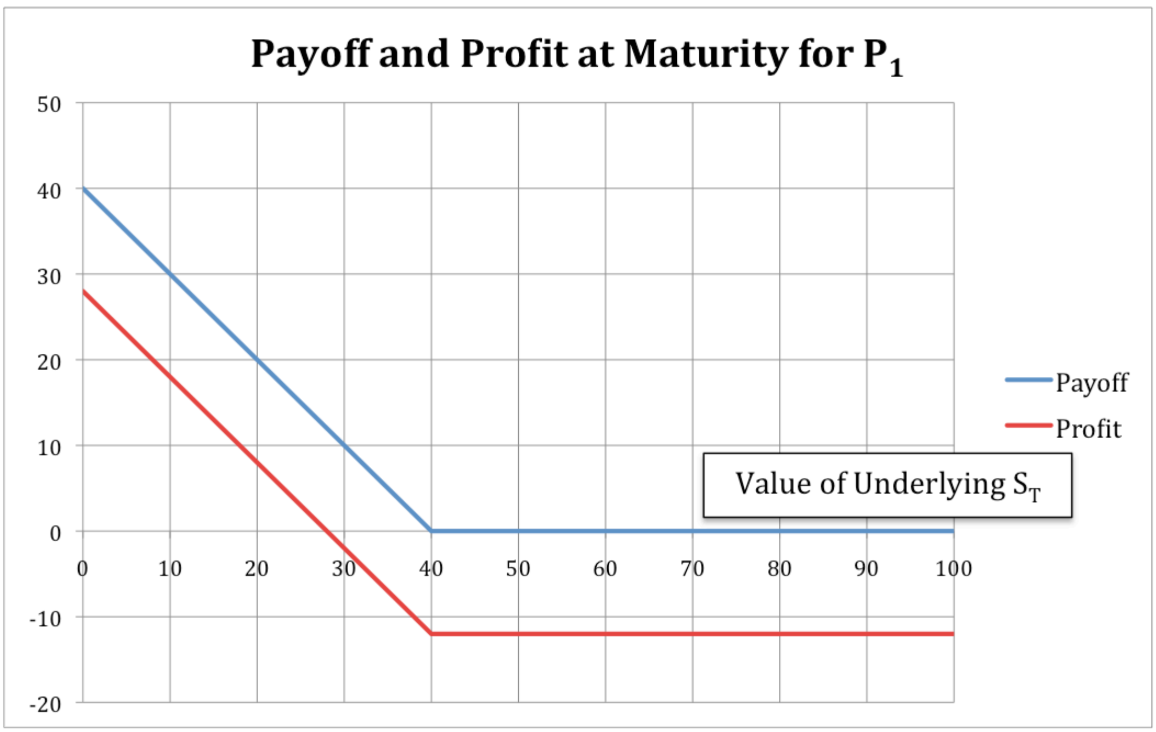
(b) The payoff of the described put option at maturity is as follows:

Underlying	**$S_T \leq 40$**	**$S_T > 40$**
Payoff Put P_1 with E=40	$40 - S_T$	0

The cost of the put option is \$12. It follows that the profit of the described put option at maturity is as follows:

Underlying	**$S_T \leq 40$**	**$S_T > 40$**
Profit Put P_1 with E=40	$28 - S_T$	-12



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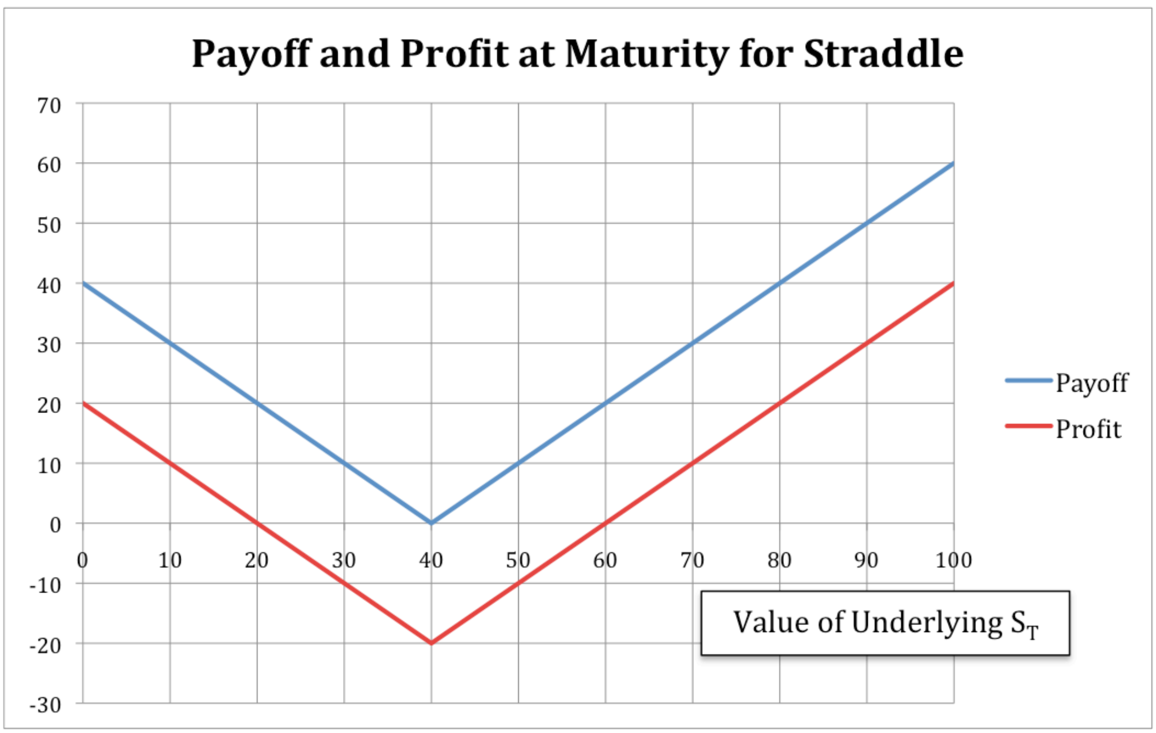
(c) The payoff of the described portfolio at maturity is obtained as follows:

Underlying	$S_T \le 40$	$S_T > 40$
Payoff Call C_1 with E=40	0	S_T-40
Payoff Put P_1 with E=40	40-S_T	0
Payoff Portfolio C_1+P_1	40-S_T	S_T-40

The portfolio C_1+P_1 costs \$8+\$12=\$20. It follows that the profit of the portfolio at maturity is as follows:

Underlying	$S_T \le 40$	$S_T > 40$
Profit Portfolio C_1+P_1	20-S_T	S_T-60



An investor might want to construct this portfolio if he thinks that the stock will move substantially between the time of construction and the maturity date, without knowing in which direction the stock will move.

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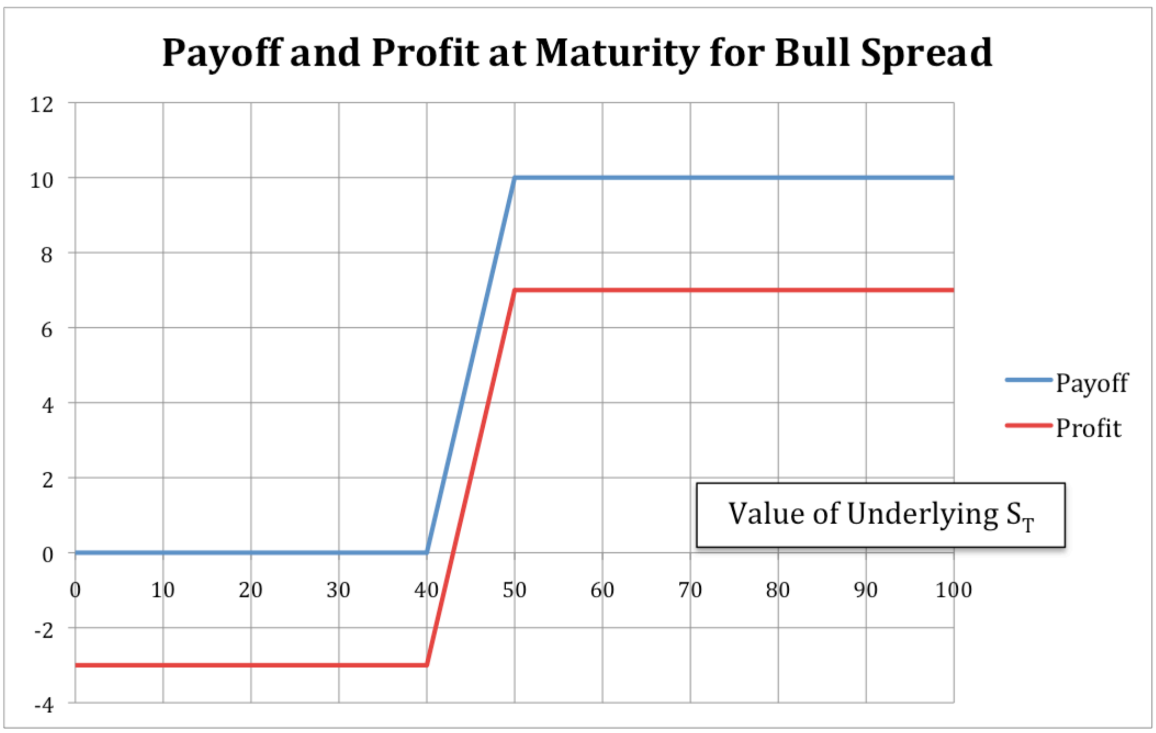
(d) The payoff of the described portfolio at maturity is obtained as follows:

Underlying	**$S_T \leq 40$**	**$40 < S_T \leq 50$**	**$S_T > 50$**
Payoff Call C_1 with E=40	0	S_T-40	S_T-40
Payoff Call C_2 with E=50	0	0	S_T-50
Payoff Portfolio C_1-C_2	0	S_T-40	10

The portfolio C_1-C_2 costs \$8-\$5=\$3. It follows that the profit of the portfolio at maturity is as follows:

Underlying	**$S_T \leq 40$**	**$40 < S_T \leq 50$**	**$S_T > 50$**
Profit Portfolio C_1-C_2	-3	S_T-43	7



An investor might want to construct this portfolio if he thinks that the price of the stock will go up moderately between the time of construction and the maturity date. Moreover, he puts a limit on his losses if the stock decreases.

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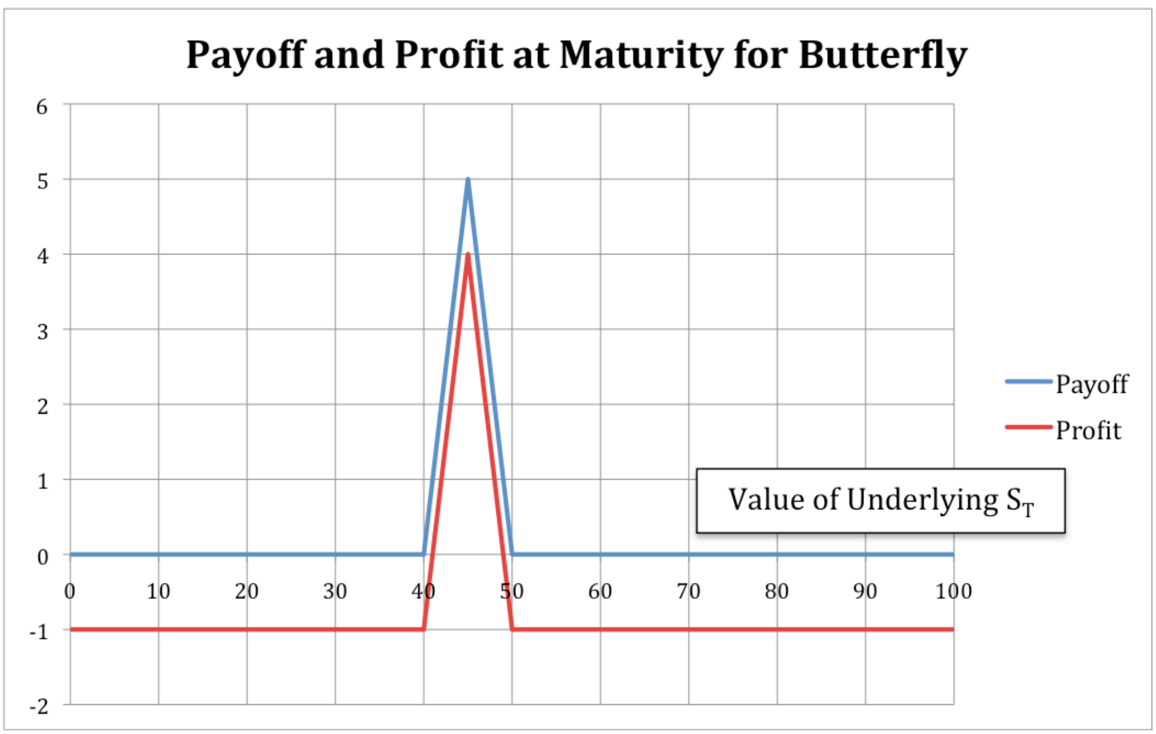
(e) The payoff and the profit of the described portfolio are as follows:

Underlying	**$S_T \le 40$**	**$40 < S_T \le 45$**	**$45 < S_T \le 50$**	**$S_T > 50$**
Payoff Call C_1 with E=40	0	S_T-40	S_T-40	S_T-40
Payoff Call C_2 with E=50	0	0	0	S_T-50
Payoff Call C_3 with E=45	0	0	S_T-45	S_T-45
Payoff Portfolio $C_1+C_2-2\bullet C_3$	0	S_T-40	50-S_T	0

The portfolio $C_1+C_2-2\bullet C_3$ costs \$8+\$5-2•\$6=\$1. It follows that the profit of the portfolio is as follows:

Underlying	**$S_T \le 40$**	**$40 < S_T \le 45$**	**$45 < S_T \le 50$**	**$S_T > 50$**
Profit Portfolio $C_1+C_2-2\bullet C_3$	-1	S_T-41	49-S_T	-1



An investor might want to construct this portfolio if he wants to bet that that the stock falls within a certain interval of prices at the maturity date. At the same, the investor's losses are limited if the stock falls outside the interval that the investor aims for.

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Question 3

The price of the underlying XYZ evolves as follows:

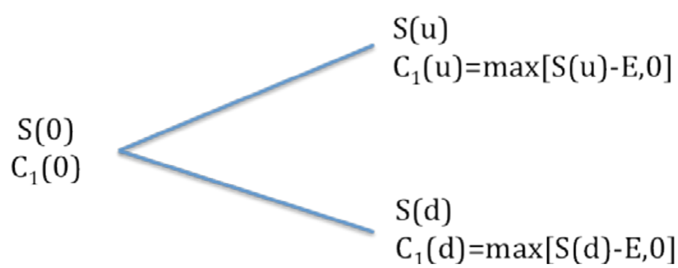
- A year from now:

  $$S(u) = u \cdot S(0) = 150, S(d) = d \cdot S(0) = 50.$$

- Two years from now:

  $$S(uu) = u^2 \cdot S(0) = 225, S(ud) = S(du) = u \cdot d \cdot S(0) = 75, S(dd) = d^2 \cdot S(0) = 25.$$

(a) The one-period Binomial Asset Pricing Model has the following schematic form:



The one-period hedge ratio for the call option C_1 is

$$H = \frac{C(u) - C(d)}{(u-d)S(0)} \Leftrightarrow H = \frac{\max[150-30,0] - \max[50-30,0]}{(1.5-0.5)\cdot 100} = 1.$$

The hedge ratio denotes the number of stocks you want to hold per option sold in order to construct a hedge portfolio, which generates the same payoffs in each state of the world. This means that the hedge portfolio is riskless. Therefore, an investor has two possibilities to transfer money from period 0 to period 1, the hedge portfolio and investing at the risk-free rate.

It then follows from the no-arbitrage principle that these two possibilities must have the same return, i.e. the rate of return for investing in the hedge portfolio must equal r. Knowing the value of the hedge portfolio in a subsequent period and the return of the hedge portfolio therefore determines the value of the call option today.

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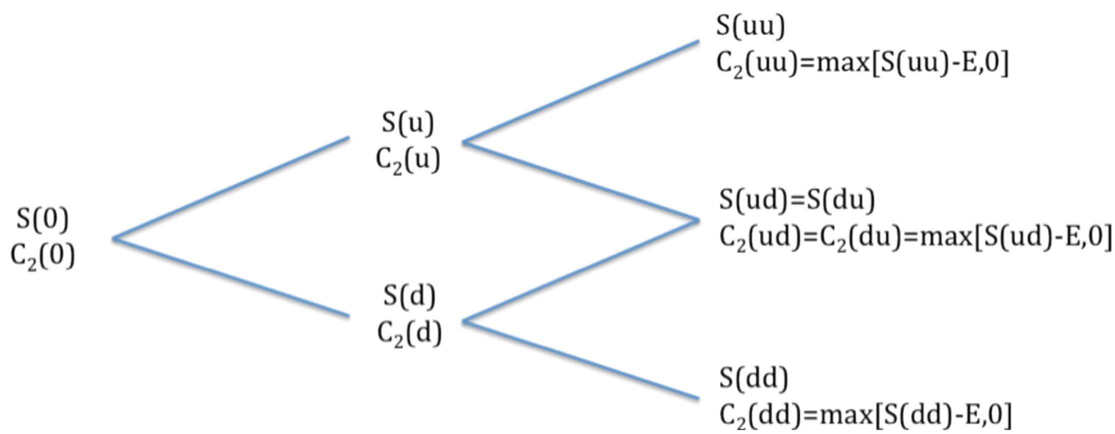
(b) The desired quantity is $C_1(0)$, which satisfies the following identity:

$$\frac{H\cdot S(u)-C_1(u)}{H\cdot S(0)-C_1(0)}=1+r \Leftrightarrow \frac{1\cdot 150-\max[150-30,0]}{1\cdot 100-C_1(0)}=1.25$$
$$\Leftrightarrow C_1(0)=76.$$

(c) The one-period Binomial Asset Pricing Model has the following schematic form:



The desired quantity $C_2(0)$ will be obtained via backward induction.

At the upper node in period 1, that is, after the stock price increases once:

$$H(u)=\frac{C_2(uu)-C_2(ud)}{(u-d)\cdot S(u)} \Leftrightarrow H(u)=\frac{\max[225-30,0]-\max[75-30,0]}{(1.5-0.5)\cdot 150}=1.$$

Furthermore, the price of C_2 at the upper node in period 1 satisfies the following identity:

$$\frac{H\cdot S(uu)-C_2(uu)}{H\cdot S(u)-C_2(u)}=1+r \Leftrightarrow \frac{1\cdot 225-\max[225-30,0]}{1\cdot 150-C_2(u)}=1.25$$
$$\Leftrightarrow C_2(u)=126.$$

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At the bottom node in period 1, that is, after the stock price increases once:

$$H(d) = \frac{C_2(du) - C_2(dd)}{(u-d)\cdot S(d)} \Leftrightarrow H(d) = \frac{\max[75-30,0] - \max[25-30,0]}{(1.5-0.5)\cdot 50} = 0.9.$$

Furthermore, the price of C_2 at the bottom node in period 1 satisfies the following identity:

$$\frac{H\cdot S(du) - C_2(du)}{H\cdot S(d) - C_2(d)} = 1 + r \Leftrightarrow \frac{0.9\cdot 75 - \max[75-30,0]}{0.9\cdot 50 - C_2(d)} = 1.25$$
$$\Leftrightarrow C_2(d) = 27.$$

Hence, $C_2(d)$=$27 is the answer to the first question.

Finally, at the initial node:

$$H(0) = \frac{C_2(u) - C_2(d)}{(u-d)\cdot S(0)} \Leftrightarrow H(0) = \frac{126-27}{(1.5-0.5)\cdot 100} = 0.99.$$

Furthermore, the price of C_2 at the initial node satisfies the following identity:

$$\frac{H\cdot S(u) - C_2(u)}{H\cdot S(0) - C_2(0)} = 1 + r \Leftrightarrow \frac{0.99\cdot 150 - 126}{0.99\cdot 100 - C_2(0)} = 1.25$$
$$\Leftrightarrow C_2(0) = 81.$$

Hence, $C_2(0)$=$81 is the answer to the second question.

(d) The price of the call-option has increased from $76 to $81, as the maturity of the call option has increased from one year to two years. This result is intuitive for call options, as a call-option becomes more valuable for higher stock prices. When the maturity becomes longer, the range of possible prices for the stock increases. The strike price of the call option however "cuts off" the bottom part of this range, leaving only the higher range of the stock price. This benefits the value of the call option.

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(e) Recall that the put-call parity is

$$C+\frac{E}{(1+r)^T}=S+P,$$

where T denotes time to maturity. It holds at any time period and for any state of the stock price evolution.

Therefore, the price of a 2-year put with the same strike price as C_2 a year from now after the price has gone down once is

$$27+\frac{30}{1+0.25}=50+P(d)\Leftrightarrow P(d)=1.$$

Analogously, the price of this put option today is:

$$81+\frac{30}{(1+0.25)^2}=100+P(0)\Leftrightarrow P(0)=0.2.$$

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Question 4

The Black-Scholes formula is

$$C = S \cdot N(d_1) - e^{-r \cdot T} \cdot E \cdot N(d_2),$$

where d_1 and d_2 are defined as follows:

$$d_1 = \frac{\ln\left(\frac{S}{E}\right) + r \cdot T + \frac{\sigma^2 \cdot T}{2}}{\sigma \cdot \sqrt{T}},$$

$$d_1 = \frac{\ln\left(\frac{S}{E}\right) + r \cdot T - \frac{\sigma^2 \cdot T}{2}}{\sigma \cdot \sqrt{T}}.$$

The notation is hereby as follows:

- C is the current price of a European call option written on a stock with strike price E and time to maturity T,
- S is the current price of the underlying stock,
- r is the risk-free interest rate,
- σ is the standard deviation of the return on the underlying stock,
- N is the cumulative normal distribution function.

(a) According to the information given, it follows that

$$d_1 = \frac{\ln\left(\frac{S}{E}\right) + r \cdot T + \frac{\sigma^2 \cdot T}{2}}{\sigma \cdot \sqrt{T}} = \frac{\ln\left(\frac{200}{120}\right) + 0.2 \cdot 4 + \frac{(0.5)^2 \cdot 4}{2}}{0.5 \cdot \sqrt{4}} \approx 1.8108,$$

$$d_2 = \frac{\ln\left(\frac{S}{E}\right) + r \cdot T - \frac{\sigma^2 \cdot T}{2}}{\sigma \cdot \sqrt{T}} = \frac{\ln\left(\frac{200}{120}\right) + 0.2 \cdot 4 - \frac{(0.5)^2 \cdot 4}{2}}{0.5 \cdot \sqrt{4}} \approx 0.8108.$$

Hence, the price of the desired call-option is given by

$$C = S \cdot N(d_1) - e^{-r \cdot T} \cdot E \cdot N(d_2) \approx 200 \cdot N(1.8108) - e^{-0.2 \cdot 4} \cdot 120 \cdot N(0.8108) \approx 150.31.$$

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(b) Recall that the put-call parity is

$$C + e^{-r \cdot T} \cdot E = S + P,$$

where the only new variable is P, which denotes the price of a put option that has the same underlying, the same strike price, and the same time to maturity as the call-option whose current price is denoted by C.

Therefore, the price of a 4-year put with the same strike price as the call option in part (a) is

$$150.31 + e^{-0.2 \cdot 4} \cdot 120 = 200 + P \Leftrightarrow P \approx 4.23.$$

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(c) According to the information given, it follows that

$$d_1 = \frac{\ln\left(\frac{S}{E}\right) + r \cdot T + \frac{\sigma^2 \cdot T}{2}}{\sigma \cdot \sqrt{T}} = \frac{\ln\left(\frac{200}{120}\right) + 0.2 \cdot 4 + \frac{(0.4)^2 \cdot 4}{2}}{0.4 \cdot \sqrt{4}} \approx 2.0385,$$

$$d_2 = \frac{\ln\left(\frac{S}{E}\right) + r \cdot T - \frac{\sigma^2 \cdot T}{2}}{\sigma \cdot \sqrt{T}} = \frac{\ln\left(\frac{200}{120}\right) + 0.2 \cdot 4 - \frac{(0.4)^2 \cdot 4}{2}}{0.4 \cdot \sqrt{4}} \approx 1.2385.$$

Hence, the price of the desired call-option is given by

$$C = S \cdot N(d_1) - e^{-r \cdot T} \cdot E \cdot N(d_2) \approx 200 \cdot N(2.0385) - e^{-0.2 \cdot 4} \cdot 120 \cdot N(1.2385) \approx 147.75.$$

According to the put-call parity, the price of a 4-year put with the same strike price as the call option above is

$$147.75 + e^{-0.2 \cdot 4} \cdot 120 = 200 + P \Leftrightarrow P \approx 1.67.$$

Both the price of the call and the put option decreases in response to a decrease of the volatility of the underlying stock.

First, consider the call option. It becomes more valuable for higher stock prices and "cuts off" stock prices below the strike price. Therefore, as stock prices are less far apart because of the lower volatility, the call option becomes less valuable.

Second, consider the put option. It becomes more valuable for lower stock prices and "cuts off" stock prices above the strike price. Therefore, as stock prices are less far apart because of the lower volatility, the put option also becomes less valuable.

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(d) According to the information given, it follows that

$$d_1 = \frac{\ln\left(\frac{S}{E}\right) + r \cdot T + \frac{\sigma^2 \cdot T}{2}}{\sigma \cdot \sqrt{T}} = \frac{\ln\left(\frac{200}{100}\right) + 0.2 \cdot 4 + \frac{(0.5)^2 \cdot 4}{2}}{0.5 \cdot \sqrt{4}} \approx 1.9931,$$

$$d_2 = \frac{\ln\left(\frac{S}{E}\right) + r \cdot T - \frac{\sigma^2 \cdot T}{2}}{\sigma \cdot \sqrt{T}} = \frac{\ln\left(\frac{200}{100}\right) + 0.2 \cdot 4 - \frac{(0.5)^2 \cdot 4}{2}}{0.5 \cdot \sqrt{4}} \approx 0.9931.$$

Hence, the price of the desired call-option is given by

$$C = S \cdot N(d_1) - e^{-r \cdot T} \cdot E \cdot N(d_2) \approx 200 \cdot N(1.9931) - e^{-0.2 \cdot 4} \cdot 100 \cdot N(0.9931) \approx 157.65.$$

According to the put-call parity, the price of a 4-year put with the same strike price as the call option above is

$$157.65 + e^{-0.2 \cdot 4} \cdot 100 = 200 + P \Leftrightarrow P \approx 2.58.$$

The price of the call increases in response to the decrease in the strike price. In contrast, the price of the put option decreases in response to the decrease in the strike price.

First, consider the call option. It becomes more valuable for higher stock prices and “cuts off” stock prices below the strike price. The decrease in the strike price therefore decreases the region for which the option “cuts off” the stock price and increases the value of the option for higher stock prices.

Second, consider the put option. It becomes more valuable for lower stock prices and “cuts off” stock prices above the strike price. The decrease in the strike price therefore increases the region for which the option “cuts off” the stock price and decreases the value of the option for lower stock prices.