Econ 252 - Financial Markets

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Problem Set 5 – Solution

Question 1

(a) In period 0, both the buyer and the seller have to post margin according to the initial margin requirement. As they enter into 100 contracts, they both have to post 100•$12=$1,200 margin.

From period 0 to period 1, the price of wheat futures that mature in period 4 falls by $4, from $130 to $126. This is disadvantageous for the buyer, as he has locked in a price of $130 in period 0. Therefore, the balance of his margin account will fall by 100•$4=$400, from $1,200 to $800. In contrast, the fact that the price of wheat futures that mature in period 4 falls by $4 is advantageous for the seller, as he has locked in a price of $130 in period 0. In consequence, the seller's balance in his margin increases from $1,200 to $1,600.

The balance of the margin accounts will evolve in periods 2 through 4 according to the same argument. The only important aspect to keep in mind is the maintenance requirement. For 100 contracts, the maintenance level equals 100•$7=$700. If any of the two party's margin account balance falls below the maintenance level, this party has to post additional margin such that the balance of the margin account is back at the initial margin level.

In consequence, the margin account balance for the buyer and the seller evolves as follows, where additions indicate additional margin postings because of balance drops below the maintenance margin:

Period	Price	Buyer's Margin Account	Seller's Margin Account
0	$130	$1,200	$1,200
1	$126	$800	$1,600
2	$120	$200+$1,000=$1,200	$2,200
3	$128	$2,000	$1,400
4	$131	$2,300	$1,100

(b) The basic logic behind the evolution of the margin accounts for the buyer and the seller remains the same as in part (a). The only additional feature in this part is that both parties always remove any balance in their respective margin accounts that exceeds the initial margin. Removal of funds will be indicated by subtractions in the following table, whereas additions still indicate additional margin that is posted because the balance of the account drops below the maintenance margin level.

Finally, note that on the settlement date neither is margin neither removed nor is additional margin posted.

Period	Price	Buyer's Margin Account	Seller's Margin Account
0	$130	$1,200	$1,200
1	$126	$800	$1,600-$400=$1,200
2	$120	$200+$1,000=$1,200	$1,800-$600=$1,200
3	$128	$2,000-$800=$1,200	$400+$800=$1,200
4	$131	$1,500	$900

(c) Period 4 is the settlement date. Therefore, the spot market price of 50 bushels of wheat must be the futures price of 50 bushels of wheat in period 4, which is $131.

Question 2

(a) The fair value of the described futures contract is given by the expression

$$(1+r-y)\bullet\text{spot price of asset X},$$

where r is the riskless interest rate and y is the dividend rate for asset X.

In consequence, the fair value of the described futures contract is

$$(1+0.03-0.02)\bullet 50=50.50.$$

(b) Now, the futures price is \$53.50. In this case, one obtains a riskless profit from the so-called "cash and carry trading strategy":

- Period 0:
  1. Sell futures contract.
  2. Borrow \$50.
  3. Use borrowed money to buy underlying asset for \$50.
- Period 1:
  1. Collect $\$1=0.02\bullet\$50$ because of the X's dividend payment.
  2. Use underlying asset to settle futures contract, receive \$53.50.
  3. Pay off loan with $\$51.50=1.03\bullet\$50$.

The total profit from these transactions is \$3.

(c) Now, the futures price is \$47.50. In this case, one obtains a riskless profit from the so-called "reverse cash and carry trading strategy":

- Period 0:
  1. Buy futures contract.
  2. Short-sell the underlying asset, and receive \$50.
  3. Lend \$50.
- Period 1:
  4. Receive \$51.50=1.03•\$50 from loan.
  5. Receive the asset from futures contract, and pay \$47.50.
  6. Return the asset from futures contract, and pay \$1=0.02•\$50 dividend to settle short-sale.

The total arbitrage profit from these transactions is \$3.

(d) Qualitatively, the fact that the futures price is above the fair value price of the futures contract results in a riskless profit opportunity (not requiring any of the investor's own capital) via the "cash and carry trading strategy". However, this strategy requires the ability to borrow money. As borrowing is prohibited, it is not possible to exploit futures prices that are too high.

In contrast, the fact that the futures price falls below the fair value price of the futures contract results in a riskless profit opportunity (not requiring any of the investor's own capital) via the "reverse cash and carry trading strategy". This strategy requires the ability to lend money, which is permitted in this part.

Therefore, futures prices that are "too low" can be exploited, whereas futures prices that are "too high" cannot be corrected. More precisely, the bound on futures prices in this part such that there are no arbitrage opportunities is

$$F \geq (1+r-y)\bullet P = (1+0.02-0.02)\bullet 50 = 50.$$

Note that in this equation, because lending is used, r = 0.02.

(e) Qualitatively, the fact that the futures price falls below the fair value price of the futures contract results in a riskless profit opportunity (not requiring any of the investor's own capital) via the "reverse cash and carry trading strategy". However, this strategy requires the ability to lend money. As lending is prohibited, it is not possible to exploit futures prices that are too low.

In contrast, the fact that the futures price is above the fair value price of the futures contract results in a riskless profit opportunity (not requiring any of the investor's own capital) via the "cash and carry trading strategy". This strategy requires the ability to borrow money, which is permitted in this part.

Therefore, futures prices that are "too high" can be exploited, whereas futures prices that are "too low" cannot be corrected. More precisely, the bound on futures prices in this part such that there are no arbitrage opportunities is

$$F \leq (1+r-y) \bullet P = (1+0.04-0.02) \bullet 50 = 51.$$

Note that in this equation, because lending is used, $r = 0.04$.

(f) Combining (d) and (e), the range of futures prices that is consistent with the no-arbitrage principle is

$$50 \leq F \leq 51.$$

Question 3

(a) The corresponding risk-weights from the Fabozzi et al. textbook are:

- U.S. Treasury securities: 0%,
- municipal general obligation bonds: 20%,
- residential mortgages: 50%,
- commercial loans and commercial mortgages: 100%.

Therefore, the risk-weighted value of these assets is (in millions of $)

$$100 \bullet 0.00 + 50 \bullet 0.20 + 500 \bullet 0.50 + 300 \bullet 1.00 = 560.$$

(b) According to the simplified capital requirement, the bank must hold 10% of the risk-weighted assets. This is $56 million.

(c) The risk-weighted value of the assets to be added is (in millions of $)

$$100 \bullet 0.20 + 200 \bullet 0.50 = 120.$$

Therefore, the risk-weighted capital requirement corresponding to these assets is $12 million, which is the smallest amount of capital it must raise to be in compliance with the simplified capital requirement.

(d) If $50 million in residential mortgage loans go bad, the total risk-weighted assets fall to (in millions of $)

$$100 \bullet 0.00 + 50 \bullet 0.20 + 450 \bullet 0.50 + 300 \bullet 1.00 = 535.$$

Therefore, the capital requirement for the bank is now $53.5 million. As the bank has just been holding enough capital to meet the capital requirement before the residential mortgages have gone bad, the bank capital before was $56 million, as computed in part (b). After losing $50 million, the bank capital falls from $56 million to $6 million.

Hence, the bank must raise at least $53.5 million – $6 million = $47.5 million to be in compliance with the simplified capital requirement.

(e) If $50 million in commercial loans go bad, the total risk-weighted assets fall to (in millions of $)

$$100 \bullet 0.00 + 50 \bullet 0.20 + 500 \bullet 0.50 + 260 \bullet 1.00 = 510.$$

Therefore, the capital requirement for the bank is now $51 million. As the bank has just been holding enough capital to meet the capital requirement before the residential mortgages have gone bad, the bank capital before was $56 million, as computed in part (b). After losing $50 million, the bank capital falls from $56 million to $6 million.

Hence, the bank must raise at least $51 million – $6 million = $45 million to be in compliance with the simplified capital requirement.