Econ 252 - Financial Markets

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Problem Set 4 – Solution

Question 1

(a) For applicant A, $$LTV_A = \frac{525{,}000}{600{,}000} = 0.875 = 87.5\%.$$

For applicant B, $$LTV_B = \frac{385{,}000}{400{,}000} = 0.9625 = 96.25\%.$$

(b) As the loan is an interest-only loan with a lockout period of two years, the outstanding balance of the loan after two years is the same as the outstanding balance at the moment of origination. So, A's mortgage balance is still \$525,000 and B's mortgage balance is still \$385,000.

With regard to A's property, the bank can sell it for
$$0.9 \cdot \$600{,}000 = \$540{,}000.$$
As the outstanding balance (\$525,000) is less than the sales price of the house (\$540,000), the bank will recover all of the originally lent money. It will even be able to distribute \$25,000 back to A.

With regard to B's property, the bank can sell it for
$$0.9 \cdot \$400{,}000 = \$360{,}000.$$
This means that the bank will only recover
$$\frac{360{,}000}{385{,}000} \approx 0.9351 = 93.51\%$$
of the originally lent amount.

(c) A high LTV exposes the bank to the risk that it will not be able to recover all the originally lent money if house prices depreciate. Therefore, the bank should have denied B's application in the first place.

(d) In order to find the outstanding balance at the end of 2 years (24 months) use the following formula:

$$MB_t = MB_0 \cdot \left[\frac{(1+i)^n - (1+i)^t}{(1+i)^n - 1} \right],$$

where

- n: number of months of the mortgage loan,
- i: note rate divided by 12,
- MB_0: original mortgage balance,
- MB_t: mortgage balance after t months.

It follows that the outstanding mortgage balance for A after 24 months is

$$MB_{t,A} = 525{,}000 \cdot \left[\frac{(1.0075)^{360} - (1.0075)^{24}}{(1.0075)^{360} - 1} \right] \approx 517{,}489.97.$$

The bank still sells A's property for $540,000. It will therefore still be able to recover all of the originally lent money. The amount that it redistributes to A is now $22,510.03.

The outstanding mortgage balance for B after 24 months is

$$MB_{t,B} = 385{,}000 \cdot \left[\frac{(1.0075)^{360} - (1.0075)^{24}}{(1.0075)^{360} - 1} \right] \approx 379{,}492.41.$$

With regard to B's property, the bank can still sell it for $360,000. This means that the bank will only recover $\dfrac{360{,}000}{379{,}492.41} \approx 0.9486 = 94.86\%,$

which is slightly higher than the recovery rate from part (b).

(e) The assessment from part (c) is unchanged.

Question 2

(a) The monthly payment is determined according to the following formula

$$MP = MB_0 \cdot \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right],$$

where

- n: number of months of the mortgage loan,
- i: note rate divided by 12,
- MB_0: original mortgage balance,
- MP: monthly mortgage payment.

It follows that

$$MP = 20{,}000 \cdot \left[\frac{0.01(1.01)^5}{(1.01)^5 - 1} \right] \approx 4{,}120.80.$$

The interest rate in a given month is computed as 0.9%, which is the note rate divided by 12, multiplied by the remaining mortgage balance in the beginning of the respective month.

Subsequently, on obtains the principal repayment in a given month from the fact that the interest payment and the principal repayment in a given month add up to the monthly mortgage payment.

In consequence, the full amortization table has the following form:

Month	**Interest Payment**	**Principal Repayment**	**Remaining Mortgage Balance in the Beginning of the Month**
1	$200.00	$3,920.80	$20,000.00
2	$160.80	$3,960.00	$16,079.20
3	$121.19	$3,999.61	$12,119.20
4	$81.20	$4,039.60	$8,119.59
5	$40.80	$4,080.00	$4,079.99

As desired, the outstanding balance at the end of the last month of the mortgage approximately equals $0. (It is not exactly equal to $0 due to rounding imprecisions in the course of the computation of the mortgage table.)

(b) The full amortization table after a prepayment of $5,000 looks as follows:

Month	Interest Payment	Principal Repayment	Remaining Mortgage Balance in the Beginning of the Month
1	$200.00	$ 3,920.80	$20,000.00
2	$160.80	$ 3,960.00	$16,079.20
3	$121.19	$5,000 + $3,999.61	$12,119.20
4	$31.20	$3,119.59	$3,119.59
5	$0.00	$0.00	$0.00

Already in month 4, the sum of the interest payment ($31.20) and the total remaining mortgage balance in the beginning of month 4 ($3,119.59) exceeds the originally agreed upon mortgage payment ($4,120.80). Therefore, the borrower pays $3,119.59+$31.20=$3,150.79 in month 4, eliminating all of the remaining mortgage balance. That is, the remaining mortgage balance in month 5 is $0, resulting in an interest payment of $0 for month 5 and a principal repayment of $0 in month 5.

(c) The monthly payment is again determined according to the formula

$$MP = MB_0 \cdot \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right],$$

where

- n: number of months of the mortgage loan,
- i: note rate divided by 12,
- MB_0: original mortgage balance,
- MP: monthly mortgage payment.

It follows that $$MP = 200{,}000 \cdot \left[\frac{0.01(1.01)^{360}}{(1.01)^{360} - 1} \right] \approx 2{,}057.23.$$

The outstanding mortgage balance in the beginning of month 100 is computed as the outstanding mortgage balance at the end of month 99 from the following formula: $$MB_t = MB_0 \cdot \left[\frac{(1+i)^n - (1+i)^t}{(1+i)^n - 1} \right]$$

where

- n: number of months of the mortgage loan,
- i: note rate divided by 12,
- MB_0: original mortgage balance,
- MB_t: remaining mortgage balance at the end of month t.

It follows that

$$MB_t = 200{,}000 \cdot \left[\frac{(1.01)^{360} - (1.01)^{99}}{(1.01)^{360} - 1} \right] = 190{,}397.42.$$

The interest rate in a given month is computed as 0.9%, which is the note rate divided by 12, multiplied by the remaining mortgage balance in the beginning of the respective month.

Subsequently, on obtains the principal repayment in a given month from the fact that the interest payment and the principal repayment in a given month add up to the monthly mortgage payment.

In consequence, the full amortization table has the following form:

Month	**Interest Payment**	**Principal Repayment**	**Remaining Mortgage Balance in the Beginning of the Month**
100	$1,903.97	$153.26	$190,397.42
101	$1,902.44	$154.79	$190,244.16
102	$1,900.89	$156.34	$190,089.37

Question 3

(a) In month 6, the CPR for 100 PSA is 6•0.2%=1.2%=0.012. Therefore, the CPR for 350 PSA is 3.5•1.2%=4.2%=0.042.

In consequence, the SMM for 350 PSA equals

$$1-(1-0.042)^{1/12} \approx 0.0036 = 0.36\%.$$

(b) In month 28, the CPR for 100 PSA is 28•0.2%=5.6%=0.056. Therefore, the CPR for 350 PSA is 3.5•5.6%=19.6%=0.196.

In consequence, the SMM for 350 PSA equals

$$1-(1-0.196)^{1/12} \approx 0.018 = 1.8\%.$$

(c) In month 248, the CPR for 100 PSA is 6%=0.06. Therefore, the CPR for 350 PSA is 3.5•6%=21%=0.21.

In consequence, the SMM for 350 PSA equals

$$1-(1-0.21)^{1/12} \approx 0.0194 = 1.94\%.$$

Question 4

(a) Using the outstanding mortgage balance in the beginning of month 180 as well as the pass-through rate, the net interest in month 180 equals

$$(7\% \cdot 120{,}000{,}000) \cdot \frac{1}{12} \approx 700{,}000.$$

The scheduled principal is the difference between the mortgage payment and the gross coupon interest. The gross coupon interest can be computed from WAC and the outstanding balance in the beginning of month 180 as follows:

$$(8\% \cdot 120{,}000{,}000) \cdot \frac{1}{12} \approx 800{,}000.$$

Therefore, one obtains the scheduled principal in month 180 as

$$1{,}900{,}000 - 800{,}000 \approx 1{,}100{,}000.$$

(b) The SMM in month 180 follows from the PSA assumption and the WAM via the CPR. Given the WAM is 345 months, the CPR in month 180 of the mortgage pass-through security for 100 PSA is 6%. Therefore, the CPR in month 180 for 150 PSA is

$$150\% \cdot 6\% = 9\% = 0.09.$$

In consequence, the SMM in month 180 for 150 PSA is

$$1 - (1 - 0.09)^{1/12} \approx 0.007828.$$

Making use of the SMM, the outstanding balance, and the scheduled principal for month 180, it follows that prepayment in month 180 equals

$$0.007828 \cdot (120{,}000{,}000 - 1{,}100{,}000) = 930{,}749.20.$$

(c) The total principal in month 180 is the sum of the scheduled principal and the prepayment in month 180, that is

$$1{,}100{,}000 + 930{,}749.20 = 2{,}030{,}749.20.$$

The total cash flow in month 180 is the sum of the net interest and the total principal in month 180, that is

$$700{,}000 + 2{,}030{,}749.20 = 2{,}730{,}749.20.$$

(d) The outstanding mortgage balance in month 181 is the difference between the outstanding mortgage balance in month 180 and the total principal in month 180, that is

$$120{,}000{,}000 - 2{,}030{,}749.20 = 117{,}969{,}250.80.$$

Question 5

(a) The total liabilities of the hypothetical non-agency CMO are give as the sum of the principal amounts for all 8 tranches. That is, the total liabilities are

$$\begin{aligned}&250{,}000{,}000 + 22{,}000{,}000 + 47{,}000{,}000 + 5{,}000{,}000\\&+17{,}000{,}000 + 9{,}000{,}000 + 14{,}000{,}000 + 40{,}000{,}000\\&= 404{,}000{,}000.\end{aligned}$$

As the liabilities are strictly less than the assets (the par value of the collateral), the CMO is overcollateralized. The amount of the overcollateralization is the difference between the value of the assets and the value of the liabilities, that is

$$410{,}000{,}000 - 404{,}000{,}000 = 6{,}000{,}000.$$

(b) The credit enhancement of Tranche 4 is the amount by which the collateral is allowed to decrease before Tranche 4 suffers its first loss. Therefore, the credit enhancement of Tranche 4 is the value of the overcollateralization plus the principal amount of Tranches 5 through 8. Explicitly, the credit enhancement of Tranche 4 equals

$$6{,}000{,}000 + 40{,}000{,}000 + 14{,}000{,}000 + 9{,}000{,}000 + 17{,}000{,}000 = 86{,}000{,}000.$$

(c) In order for tranche 2 to lose any value, Tranches 3 through 8 in addition to the overcollateralization must be depleted entirely. This will be the case when the value of the collateral drops by

$$\begin{aligned}&6{,}000{,}000 + 40{,}000{,}000 + 14{,}000{,}000 + 9{,}000{,}000\\&+17{,}000{,}000 + 5{,}000{,}000 + 47{,}000{,}000\\&= 138{,}000{,}000.\end{aligned}$$

So, Tranche 2 loses half of its value when the value of the collateral drops by

$$138{,}000{,}000 + \frac{1}{2}\cdot 22{,}000{,}000 = 149{,}000{,}000.$$