Econ 252 - Financial Markets

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Problem Set 3 – Solution

Question 1

The relevant formula for a coupon bond is

$$P = C \cdot \left(\frac{1 - \frac{1}{(1+\gamma)^n}}{\gamma} \right) + \frac{M}{(1+\gamma)^n},$$

with the following notation:

- P: price of the coupon bond contract today,
- C: coupon payment in each period,
- γ: yield referring to one period,
- M: maturity value,
- n: number of periods.

(a) A period refers to a year. That is, n=25.

The problem states that M=$10,000.

As a period refers to a year, γ=5%=0.05, and the coupon payment C equals 7% of $10,000, which is $700.

It follows that the price of the bond is given by

$$700\cdot\left(\frac{1-\dfrac{1}{(1.05)^{25}}}{0.05}\right)+\frac{10,000}{(1.05)^{25}}\approx 12,818.79.$$

(b) A period still refers to a year. That is, n=25.

The problem states that M=$10,000.

As a period refers to a year, γ=8%=0.08, and the coupon payment C again equals 7% of $10,000, which is $700.

It follows that the price of this bond is given by

$$700\cdot\left(\frac{1-\dfrac{1}{(1.08)^{25}}}{0.08}\right)+\frac{10,000}{(1.08)^{25}}\approx 8,932.52.$$

(c) Now, a period refers to 6 months. It follows that n=50.

As before, M=$10,000.

First, consider the bond contract from part (a):

Because a period is half a year, the relevant yield is γ=0.5•5%=2.5%=0.025. Analogously, the coupon payment satisfies C=0.5•7%•$10,000=$350.

So, the price of the bond contract in part (a) with semiannual coupon payments equals

$$350 \cdot \left(\frac{1 - \frac{1}{(1.025)^{50}}}{0.025} \right) + \frac{10{,}000}{(1.025)^{50}} \approx 12{,}836.23.$$

When the coupon payments for the bond contract in part (b) are made semiannually, one obtains the relevant yield as γ=0.5•8%=4%=0.04. The coupon payments remains at C=0.5•7%•$10,000=$350.

So, the price of the bond contract in part (a) with semiannual coupon payments equals

$$350 \cdot \left(\frac{1 - \frac{1}{(1.04)^{50}}}{0.04} \right) + \frac{10{,}000}{(1.04)^{50}} \approx 8{,}925.89.$$

(d) The relevant formula for a zero-coupon bond is

$$P = \frac{M}{(1+\gamma)^n},$$

with the following notation:

- P: price of the coupon bond contract today,
- γ: yield referring to one period,
- M: maturity value,
- n: number of periods.

As in part (a), the relevant time period is one year. Taking into account that the zero-coupon contract has the same maturity, the same yield, and the same price as the coupon bond, it follows that

$$12{,}818.79 = \frac{M}{(1.05)^{25}} \Leftrightarrow M \approx 43{,}408.97.$$

Question 2

(a) The relevant formula for a consol is

$$P = \frac{x}{\gamma},$$

with the following notation:

- P: price of the consol today,
- x: payment of the consol per period,
- γ: yield referring to one period.

As a period refers to one year, x=$50 and γ=3.5%=0.035, implying

$$P = \frac{50}{0.035} \approx 1{,}428.57.$$

(b) In this part, a period refers to six months. It follows that x=$25 and that γ=0.5•3.5%=1.75%=0.0175. Therefore, the price of the consol is now

$$P = \frac{25}{0.0175} \approx 1{,}428.57,$$

which is identical to the result in part (a).

(c) The relevant formula for an annuity is

$$P = x \cdot \left(\frac{1 - \frac{1}{(1+\gamma)^n}}{\gamma} \right),$$

with the following notation:

- P: price of the annuity today,
- x: payment of the annuity per period,
- γ: yield referring to one period,
- n: number of periods.

As a period refers to one year, n=30. Therefore, x=$50 and γ=3.5%=0.035, so that one obtains

$$P = 50 \cdot \left(\frac{1 - \frac{1}{(1.035)^{30}}}{0.035} \right) \approx 919.60.$$

(d) In this part, a period refers to six months, implying that n=2•30=60. It follows that x=$25 and that γ=0.5•3.5%=1.75%=0.0175. Therefore, the price of the annuity is now

$$25 \cdot \left(\frac{1 - \frac{1}{(1.0175)^{60}}}{0.0175} \right) \approx 924.10.$$

Question 3

The relevant formula for a coupon bond is

$$P = C \cdot \left(\frac{1 - \frac{1}{(1+\gamma)^n}}{\gamma} \right) + \frac{M}{(1+\gamma)^n},$$

with the following notation:

- P: price of the coupon bond contract today,
- C: coupon payment in each period,
- γ: yield referring to one period,
- M: maturity value,
- n: number of periods.

As stated in the problem, M=$500.

A period always refers to a six months. Therefore, the coupon payment in each period equals 0.5•4%•$500=$10.

(a) In this part, n=2•10=20.

As the price of the coupon bond is higher than the principal value, the yield of the bond cannot be higher than 4%, which is the coupon rate. So, the yield cannot be equal to 4.5%.

In order to check whether the yield is equal to 2% or 3.5%, it is sufficient to check one of the two yield values, as the problem states that one of the yield values is the correct answer.

Applying the price formula with γ=0.5•3.5%=1.75%=0.0175, one obtains

$$10\cdot\left(\frac{1-\frac{1}{(1.0175)^{20}}}{0.0175}\right)+\frac{500}{(1.0175)^{20}}\approx 520.94.$$

Therefore, 3.5% is the approximately correct yield.

If one applied the price formula with γ=0.5•2%=1%=0.01, one would obtain

$$10\cdot\left(\frac{1-\frac{1}{(1.01)^{20}}}{0.01}\right)+\frac{500}{(1.01)^{20}}\approx 590.23>520.94.$$

This shows that 2% is not the correct yield.

(b) In this part, n=2•5=10.

As the price of the coupon bond is higher than the principal value, the yield of the bond cannot be higher than 4%, which is the coupon rate. So, the yield cannot be equal to 5%.

In order to check whether the yield is equal to 1.5% or 3%, it is sufficient to check one of the two yield values, as the problem states that one of the yield values is the correct answer.

Applying the price formula with γ=0.5•1.5%=0.75%=0.0075, one obtains

$$10\cdot\left(\frac{1-\frac{1}{(1.0075)^{10}}}{0.0075}\right)+\frac{500}{(1.0075)^{10}}\approx 559.99$$

Therefore, 1.5% is the correct yield.

If one applied the price formula with γ=0.5•3%=1.5%=0.015, one would obtain

$$10\cdot\left(\frac{1-\frac{1}{(1.015)^{10}}}{0.015}\right)+\frac{500}{(1.015)^{10}}\approx 523.06<559.99.$$

This shows that 3% is not the correct yield.

(c) In this part, n=2•15=30.

As the price of the coupon bond is lower than the principal value, the yield of the bond cannot be lower than 4%, which is the coupon rate. So, the yield cannot be equal to 1% or 4%.

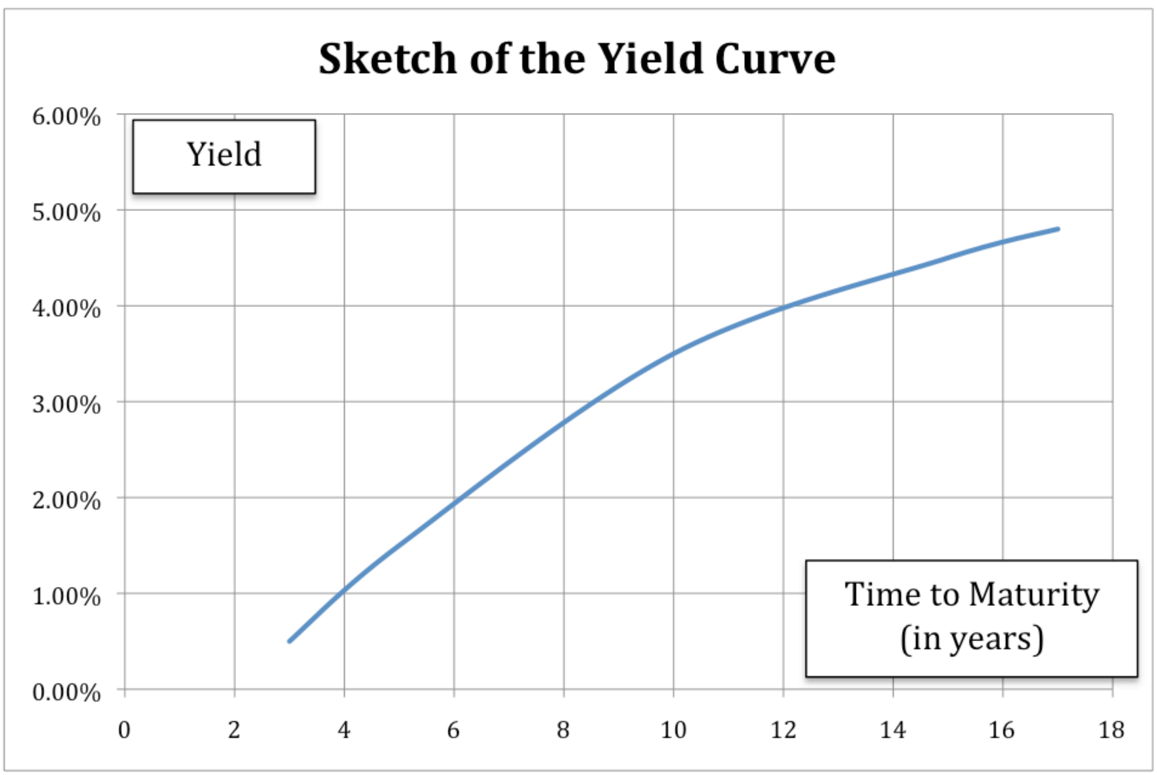
So, one already knows that the yield is equal to 4.5%.

Applying the price formula with γ=0.5•4.5%=2.25%=0.0225, one obtains

$$10\cdot\left(\frac{1-\frac{1}{(1.0225)^{30}}}{0.0225}\right)+\frac{500}{(1.0225)^{20}}\approx 472.94$$

Therefore, 4.5% is indeed the correct yield.

(d)



The displayed yield curve is positive.

Question 4

Throughout this problem, a period corresponds to 6 months.

(a) Denote the one-period forward rate between 6 and 18 months by f.

The 6-month spot rate for one period is 0.5•4%=2%=0.02. The 18-month spot rate for one period is 0.5•6%=3%=0.03.

The time between 6 months and 18 months consists of two periods.

It follows that the one-period forward rate between 6 and 18 months satisfies

$$(1+f)^2 = \frac{(1.03)^3}{1.02} \approx 1.0713,$$

implying that f≈3.5%.

In consequence, the annualized forward rate between 6 and 18 months is 2•3.5%=7%.

(b) Consider the following situation:

- You collect \$a 6 months from now.
- You would like to invest the money collected 6 months from now for 12 months, i.e. you will collect the proceeds from the investment in 18 months from now.
- You are only allowed to use today's spot rates to perform the investment.

As in part (a), the 6-month spot rate for one period is 0.5•4%=2%=0.02. The 18-month spot rate for one period is 0.5•6%=3%=0.03.

The investment strategy is as follows:

- Borrow \$a/1.02 today at the 6-month spot rate, to be repaid 6 months from now.
- Invest the borrowed money at the 18-month spot rate for 18 months.

The balance of this investment strategy evolves as follows:

- Today:
  You invest all of the borrowed money. That is, you do not need to put up any of your own capital and there is no capital left over at the end of today.
- 6 months from now:
  The 6-month loan matures. So, you need to repay the originally borrowed amount multiplied by the 6-month spot rate for one period, that is,
  $$\frac{a}{1.02} \cdot 1.02 = a.$$
  You use the \$a that you collect 6 months from now to repay the loan.
  In summary, you start with \$a and you have no capital left at the end of 6 months from now.

- 18 months from now:

  The 18-month investment matures. So, you collect the invested amount multiplied by the third power of the 18-month spot rate for one period. The third power originates from the fact that three periods pass between today and 18 months from now. That is,

$$\frac{a}{1.02} \cdot (1.03)^3 = a \cdot \frac{(1.03)^3}{1.02}.$$

  So, you start with no money at all and you have $a • $(1.03)^3/1.02$ at the end of 18 months from now.

In summary, the investment strategy turns $a into $a • $(1.03)^3/1.02$. The factor $(1.03)^3/1.02$ exactly corresponds to the factor $(1+f)^2$ from part (a), which gives rise to an annualized forward rate between 6 and 18 months of approximately 7%.

(c) The Pure Expectation Theory states that the expected 12-month spot rate 6 months from now is equal to the forward rate between 6 and 18 months (that is, between 6 months and 12 months later than that). Therefore, the expected 12-month spot rate 6 months from now is 7% according to the Pure Expectations Theory.

(d) The Liquidity Theory states that spot rates might involve a liquidity premium, which increases for longer time horizons. According to this theory, investors demand this premium in order to be compensated for the risks involved in holding longer-term maturities.

The presence of the liquidity premia affects the computation of forward rates, as the forward rates will then reflect both interest rate expectations and some manifestation of the liquidity premia. In consequence, forward rates are no longer an unbiased expectation of future spot rates, which is the key prediction out of the Pure Expectations Hypothesis.

(e) Denote the one-period forward rate between 6 months and 5 years (60 months) by f.

The 6-month spot rate for one period is 0.5•4%=2%=0.02. The 60-month spot rate for one period is 0.5•8%=4%=0.04.

The time between 6 months and 60 months consists of nine periods.

It follows that the one-period forward rate between 6 and 60 months satisfies

$$(1+f)^9 = \frac{(1.04)^{10}}{1.02} \approx 1.4512,$$

implying that f≈4.22%.

In consequence, the annualized forward rate between 6 and 60 months is 2•4.22%=8.44%.

(f) Consider the following situation:

- You collect $a 6 months from now.
- You would like to invest the money collected 6 months from now for 54 months, i.e. you will collect the proceeds from the investment in 60 months from now.
- You are only allowed to use today's spot rates to perform the investment.

As in part (e), the 6-month spot rate for one period is 0.5•4%=2%=0.02. The 60-month spot rate for one period is 0.5•8%=4%=0.04.

The investment strategy is as follows:

- Borrow $a/1.02 today at the 6-month spot rate, to be repaid 6 months from now.
- Invest the borrowed money at the 60-month spot rate for 60 months.

The balance of this investment strategy evolves as follows:

- Today:

  You invest all of the borrowed money. That is, you do not need to put up any of your own capital and there is no capital left over at the end of today.
- 6 months from now:

  The 6-month loan matures. So, you need to repay the originally borrowed amount multiplied by the 6-month spot rate for one period, that is,

  $$\frac{a}{1.02} \cdot 1.02 = a.$$

  You use the $a that you collect 6 months from now to repay the loan.

  In summary, you start with $a and you have no capital left at the end of 6 months from now.

- 18 months from now:

  The 18-month investment matures. So, you collect the invested amount multiplied by the ninth power of the 60-month spot rate for one period. The third power originates from the fact that nine periods pass between today and 60 months from now. That is,

$$\frac{a}{1.02} \cdot (1.04)^{10} = a \cdot \frac{(1.04)^{10}}{1.02}.$$

  So, you start with no money at all and you have \$a • $(1.04)^{10}/1.02$ at the end of 60 months from now.

In summary, the investment strategy turns \$a into \$a • $(1.04)^{10}/1.02$. The factor $(1.04)^{10}/1.02$ exactly corresponds to the factor $(1+f)^9$ from part (a), which gives rise to an annualized forward rate between 6 and 60 months of approximately 8.44%.

(g) Denote the one-period forward rate between 18 months and 5 years (60 months) by f.

The 18-month spot rate for one period is 0.5•6%=3%=0.03. The 60-month spot rate for one period is 0.5•8%=4%=0.04.

The time between 18 months and 60 months consists of seven periods.

It follows that the one-period forward rate between 18 and 60 months satisfies

$$(1+f)^7 = \frac{(1.04)^{10}}{(1.03)^3} \approx 1.3546,$$

implying that f≈4.43%.

In consequence, the annualized forward rate between 18 and 60 months is 2•4.43%=8.86%.

Question 5

(a) The value of X's shareholder equity is the difference between the value of X's assets minus the value of X's liabilities. According to X's last quarterly filing, the value of its assets is \$100,000,000 and the value of its liabilities is \$70,000,000. Therefore, the value of its shareholder equity is

$$\$100{,}000{,}000 - \$70{,}000{,}000 = \$30{,}00{,}000.$$

(b) Market capitalization is defined as the product of the number of shares and the share price. It follows that X's market capitalization equals

$$50{,}000{,}000 \cdot \$15 = \$75{,}000{,}000.$$

(c) The market capitalization substantially exceeds the value of the shareholder equity. There is therefore a lot of value in keeping the firm operating. In consequence, buying all shares of Corporation X in order to liquidate it is not reasonable.

(d) Issuing the bond contract worth \$1,000,000 increases X's liabilities by exactly this amount. However, the money collected from the issuance is now cash that is available to the firm. That is, X's assets also increase by \$1,000,000. Finally, X's equity is completely unaffected by the described bond issuance.

In summary, the new values are as follows:

- Assets: \$101,000,000,
- Liabilities: \$71,000,000,
- Equity: \$30,000,000.

(e) Issuing the new shares worth \$5,000,000 increases X's equity by exactly this amount. However, the money collected from the issuance is now cash that is available to the firm. That is, X's assets also increase by \$1,000,000. Finally, X's liabilities are completely unaffected by the described share issuance.

In summary, the new values are as follows:

- Assets: \$105,000,000,
- Liabilities: \$70,000,000,
- Equity: \$35,000,000.