## Spring 2011

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## Midterm Exam \#2 - Practice Exam <br> Suggested Solution

## Part I.

1. Fabozzi et al., pp. 194-195.
"The real rate is the growth in the power to consume over the life of a loan. The nominal rate of interest, by contrast, is the number of monetary units to be paid per unit borrowed, and is, in fact, the observable market rate on a loan."

The exact relation between the real and the nominal rate of interest is

$$
(1+i)=(1+r) \cdot(1+p),
$$

where is the nominal interest rate, $r$ is the real interest rate, and $p$ is the expected inflation percentage.
2. Fabozzi et al., p. 427-428.
"The Eurobond sector of the global bond market includes bonds with several distinguishing features:

- they are underwritten by an international syndicate,
- at issuance they are offered simultaneously to investors in a number of countries,
- they are issued outside the jurisdiction of any single country, and
- they are in unregistered form.

Although Eurobonds are typically listed on a national stock exchange (the most common are the Luxembourg, London, or Zurich exchanges), the bulk of all trading is in the over-thecounter market."
3. Fabozzi et al., pp. 512-515.

- Credit card receivables,
- auto loans,
- state-legislative approved special charges, charged by utility companies,
- student loans,
- Small Business Administration loans.

4. Fabozzi et al., p. 468.

People will prepay if interest rates decline, and so the investor loses the higher interest rate, which is disadvantageous for the investor.
5. Shiller manuscript, chapter 3.
"Henry David Thoreau (1817-62) wrote a classic book Walden about his dropping completely out of modern society at Walden Pond 1845-7, and contemplating nature and spirituality. But he was not really an advocate of dropping out, and in fact in his own life he did not. Throughout most of his life he was actually involved in managing his family's pencil company. He even invented a new way of making pencil leads better. So, he was actually an inspired businessman. He just thought that making money should never be the overriding purpose. 'To have done anything by which you earned money merely is to have been truly idle or worse.' The key word in this sentence is 'merely.' And we have noted that so most of our Forbes 400 are not merely in finance; they have other purposes."
6. Lecture 12 on "Misbehavior, Crises, Regulation, and Self Regulation."

Blue Sky laws attempted to curtail dishonest sales techniques for securities. Mostly state governments enacted them in the early 20th century, during the progressive era. Their principal shortcoming was that they did not deal well with businesses that crossed state lines.
7. Lecture 11 on "Behavioral Finance and the Role of Psychology."

Insurance companies might be inclined to push insuring small salient risks, such as laptop insurance or wedding ring insurance over health or life insurance.
8. Lecture 10 on "Real Estate"; Fabozzi et al., pp. 490-493.

Rating agencies are said to have been in effect "bribed" by their clients to issue high ratings on questionable securities, thereby lulling people into false sense of safety, thus causing the crashes.
9. Lecture 10 on "Real Estate."

Amortizing mortgages have a constant payment every month, and then just end, while balloon mortgages have a large payment at the end. Often balloon mortgages are short-term, and need to be refinanced regularly. People might not be able to refinance if economic hardship strikes. Also, they may not plan well enough to save to make the balloon payment at the end.
10. Lecture 9 on "Corporate Stocks."

Price routinely drops on that date, by approximately the amount of the dividend. It doesn't matter, since you get the dividend. (People in certain tax brackets may think about tax effects.) Stockbrokers should not "sell dividends," meaning to urge investors to buy a stock just before a dividend is paid, or sell just after it is paid.

## Part II.

## Question 1

The relevant formula for a coupon bond is

$$
P=C \cdot\left(\frac{1-\frac{1}{(1+\gamma)^{n}}}{\gamma}\right)+\frac{M}{(1+\gamma)^{n}},
$$

with the following notation:

- P: price of the coupon bond contract today,
- C: coupon payment in each period,
- $\gamma$ : yield referring to one period,
- M: maturity value,
- n : number of periods.
(a) A period refers to a year. That is, $\mathrm{n}=10$.

The problem states that $M=\$ 125,000$.

As a period refers to a year, $\gamma=5 \%=0.05$, and the coupon payment $C$ equals $2.5 \%$ of $\$ 125,000$, which is $\$ 3,125$.

It follows that the price of the bond is given by

$$
3,125 \cdot\left(\frac{1-\frac{1}{(1.05)^{10}}}{0.05}\right)+\frac{125,000}{(1.05)^{10}} \approx 100,869.58
$$

(b) Now, a period refers to 6 months. It follows that $\mathrm{n}=20$.

As before, $\mathrm{M}=\$ 125,000$.

Because a period is half a year, the relevant yield is $\gamma=0.5 \cdot 5 \%=2.5 \%=0.025$. Analogously, the coupon payment satisfies $\mathrm{C}=0.5 \cdot 2.5 \% \bullet \$ 125,000=\$ 1,562.50$.

So, the price of the bond contract in part (a) with semiannual coupon payments equals

$$
1,562.50 \cdot\left(\frac{1-\frac{1}{(1.025)^{20}}}{0.025}\right)+\frac{125,000}{(1.025)^{20}}=100,641.93
$$

## Question 2

As stated, a time period refers to six months.
(a) It is possible to compute the 30 -month spot rate.

Denote the one-period quantity of the desired 30-month spot rate by z.

The 72 -month spot rate for one period is $0.5 \cdot 4.5 \%=2.25 \%=0.0225$. The forward rate between 30 and 72 months for one period is $0.5 \cdot 3.5 \%=1.75 \%=0.0175$.

30 months corresponds to 5 periods and 72 months to 12 periods. In consequence, there are 7 period between 30 and 72 months.

One therefore obtains that z satisfies the following identity:

$$
(1.0175)^{7}=\frac{(1.0225)^{12}}{(1+z)^{5}}
$$

This implies that $\mathrm{z} \approx 2.95 \%$. That is, the annualized 30 -month spot rate equals approximately 5.9\%.
(b) As in part (a), the 72 -month spot rate for one period is $0.5 \cdot 4.5 \%=2.25 \%=0.0225$. Moreover, the 30 -month spot rate for one period has been computed as $0.5 \bullet 5.9 \%=2.95 \%=0.0295$.

The investment strategy is as follows:

- Borrow $\$ a /(1.0295)^{5}$ today at the 30 -month spot rate, to be repaid 30 months from now.
- Invest the borrowed money at the 72-month spot rate for 72 months.

The balance of this investment strategy evolves as follows:

- Today:

You invest all of the borrowed money. That is, you do not need to put up any of your own capital and there is no capital left over at the end of today.

- 30 months from now:

The 30 -month loan matures. So, you need to repay the originally borrowed amount multiplied by the 30 -month spot rate for five periods, that is,

$$
\frac{a}{(1.0295)^{5}} \cdot(1.0295)^{5}=a
$$

You use the \$a that you collect 30 months from now to repay the loan.
In summary, you start with \$a and you have no capital left at the end of 30 months from now.

- 72 months from now:

The 72-month investment matures. So, you collect the invested amount multiplied by the twelfth power of the 72 -month spot rate for one period. The twelfth power originates from the fact that twelve periods pass between today and 72 months from now. That is,

$$
\frac{a}{(1.0295)^{5}} \cdot(1.0225)^{12}=a \cdot \frac{(1.0225)^{12}}{(1.0295)^{5}} .
$$

So, you start with no money at all and you have $\$$ a $\bullet(1.0225)^{12} /(1.0295)^{5}$ at the end of 72 months from now.

In summary, the investment strategy turns \$a into \$a • $(1.0225)^{12} /(1.0295)^{5}$. The factor $(1.0225)^{12} /(1.0295)^{5}$ exactly corresponds to the factor $(1.0175)^{7}$, which gives rise to an annualized forward rate between 30 and 72 months of $3.5 \%$.
(c) The Pure Expectation Theory states that the expected 42-month spot rate 30 months from now is equal to the forward rate between 30 and 72 months (that is, between 30 months and 42 months later than that). Therefore, the expected 42-month spot rate 30 months from now is $3.5 \%$ according to the Pure Expectations Theory.

## Question 3

(a) The loan-to-value ratio (LTV) is

$$
L T V=\frac{425,000}{550,000}=0.7727=77.27 \%
$$

The LTV is one metric that captures the riskiness of a loan. The higher the LTV, the lower the amount by which the value of the underlying house can decrease, before the bank loses money on the mortgage. That is, the higher the LTV, the higher the risk of the mortgage for the bank.
(b) The monthly payment is determined according to the formula

$$
M P=M B_{0} \cdot\left\lfloor\frac{i(1+i)^{n}}{(1+i)^{n}-1}\right\rfloor
$$

where

- n: number of months of the mortgage loan, that is $n=12 \bullet 15=180$,
- i: note rate divided by 12 , that is $\mathrm{i}=10.8 \% / 12=0.9 \%=0.009$,
- $\mathrm{MB}_{0}$ : original mortgage balance, that is $\mathrm{MB}_{0}=425,000$,
- MP: monthly mortgage payment.

It follows that

$$
M P=425,000 \cdot\left\lfloor\frac{0.009(1.009)^{180}}{(1.009)^{180}-1}\right\rfloor \approx 4,777.30
$$

The outstanding mortgage balance in the beginning of month 91 is computed as the outstanding mortgage balance at the end of month 90 from the following formula:

$$
M B_{t}=M B_{0} \cdot\left\lfloor\frac{(1+i)^{n}-(1+i)^{t}}{(1+i)^{n}-1}\right\rfloor
$$

where

- n : number of months of the mortgage loan, that is $\mathrm{n}=12 \cdot 15=180$,
- i: note rate divided by 12 , that is $\mathrm{i}=10.8 \% / 12=0.9 \%=0.009$,
- $\mathrm{MB}_{0}$ : original mortgage balance, that is $\mathrm{MB}_{0}=425,000$,
- $\mathrm{MB}_{\mathrm{t}}$ : remaining mortgage balance at the end of month t .

It follows that $M B_{90}=425,000 \cdot\left\lfloor\frac{(1.009)^{180}-(1.009)^{90}}{(1.009)^{180}-1}\right\rfloor \approx 293,818.17$.

The interest rate in a given month is computed as $0.9 \%$, which is the note rate divided by 12 , multiplied by the remaining mortgage balance in the beginning of the respective month.

Subsequently, on obtains the principal repayment in a given month from the fact that the interest payment and the principal repayment in a given month add up to the monthly mortgage payment.

In consequence, the full amortization table has the following form:

| Month | Interest <br> Payment | Principal <br> Repayment | Remaining Mortgage <br> Balance in the <br> Beginning of the Month |
| :---: | ---: | ---: | ---: |
| 91 | $\$ 2,644.36$ | $\$ 2,132.94$ | $\$ 293,818.17$ |
| 92 | $\$ 2,625.17$ | $\$ 2,152.13$ | $\$ 291,685.23$ |

## Question 4

(a) The credit enhancement (provided by the senior-subordinate structure) of tranche 5 is the sum of the principal amounts for tranches 6 through 8 plus the overcollateralization. As the sum of the principal amounts for tranches 6 through 8 equals $\$ 65,000,000$, it follows from the credit enhancement being equal to $\$ 90,000,000$ that the overcollateralization equals

$$
\$ 90,000,000-\$ 65,000,000=\$ 25,000,000 .
$$

(b) Given that the overcollateralization equals $\$ 25,000,000$, it follows that the sum of all principal amounts in the CMO needs to equal
\$600,000,000 - \$25,000,000 = \$575,000,000.

The sum of the principal amounts of all tranches besides tranche 4 equals $\$ 530,000,000$. Therefore, the principal amount of tranche 4 equals $\$ 45,000,000$.

The credit enhancement (provided by the senior-subordinate structure) of tranche 2 is the sum of the principal amounts of tranches 3 through 8 plus the overcollateralization. It is therefore equal to $\$ 220,000,000$.

If $\$ 30,000,000$ is used for the overcollateralization as specified in the hint, the results are $\$ 40,000,000$ for the principal amount of tranche 4 and still $\$ 220,000,000$ for the credit enhancement of tranche 2 .

