(c) In order to compute the amortization table for months 70 and 71, one first needs to compute the remaining mortgage balance in the beginning of month 70. This balance is computed as the outstanding mortgage balance at the end of month 69 from the following formula:

$$MB_t = MB_0 \cdot \left[\frac{(1+i)^n - (1+i)^t}{(1+i)^n - 1} \right],$$

where

- n: number of months of the mortgage loan, that is n=12•15=180,
- i: note rate divided by 12, that is i=7.2%/12=0.6%=0.006,
- MB_0: original mortgage balance, that is MB_0=335,000,
- MB_t: remaining mortgage balance at the end of month t.

It follows that

$$MB_{69} = 335{,}000 \cdot \left[\frac{(1.006)^{180} - (1.006)^{69}}{(1.006)^{180} - 1} \right] \approx 246{,}543.37.$$

The interest rate in a given month is computed as 0.6%, which is the note rate divided by 12, multiplied by the remaining mortgage balance in the beginning of the respective month.

Subsequently, on obtains the principal repayment in a given month from the fact that the interest payment and the principal repayment in a given month add up to the monthly mortgage payment.

In consequence, the full amortization table has the following form:

Month	**Interest Payment**	**Principal Repayment**	**Remaining Mortgage Balance in the Beginning of the Month**
70	$1,479.26	$1,569.40	$246,543.37
71	$1,469.84	$1,578.82	$244,973.97

Question 3

(a) The value of X's shareholder equity is the difference between the value of X's assets minus the value of X's liabilities. According to X's last quarterly filing, the value of its assets is $250,000,000 and the value of its liabilities is $200,000,000. Therefore, the value of its shareholder equity is

$$\$250{,}000{,}000 - \$200{,}000{,}000 = \$50{,}00{,}000.$$

(b) Market capitalization is defined as the product of the number of shares and the share price. It follows that X's market capitalization equals

$$20{,}000{,}000 \cdot \$6 = \$120{,}000{,}000.$$

The market capitalization substantially exceeds the value of the shareholder equity. There is therefore a lot of value in keeping the firm operating. In consequence, buying all shares of Corporation X in order to liquidate it is not reasonable.

(d) Issuing the bond contract worth $1,000,000 increases X's liabilities by exactly this amount. However, the money collected from the issuance is now cash that is available to the firm. That is, X's assets also increase by $1,000,000. Finally, X's equity is completely unaffected by the described bond issuance.

Issuing the new shares worth $4,000,000 increases X's equity by exactly this amount. The money collected from the issuance is now cash that is available to the firm. That is, X's assets increase by another $4,000,000. Finally, X's liabilities are completely unaffected by the described share issuance.

In summary, the new values are as follows:

- Assets: $255,000,000,
- Liabilities: $201,000,000,
- Equity: $54,000,000.

Question 4

(a) The dividend payment of $6 for asset X translates into a dividend rate of 4%, as 6/200=0.03.

The fair value of the described futures contract is given by the expression

(1+r-y)•spot price of asset X,

where r is the riskless interest rate and y is the dividend rate for asset X.

In consequence, the fair value of the described futures contract is

(1+0.04-0.03)•200=202.

Therefore, when the actual futures price is $210, one would necessarily have to implement the "cash and carry trading strategy." However, this strategy entails purchasing asset Y, which the investor is contractually barred to do. Therefore, it is not possible to exploit this arbitrage opportunity.

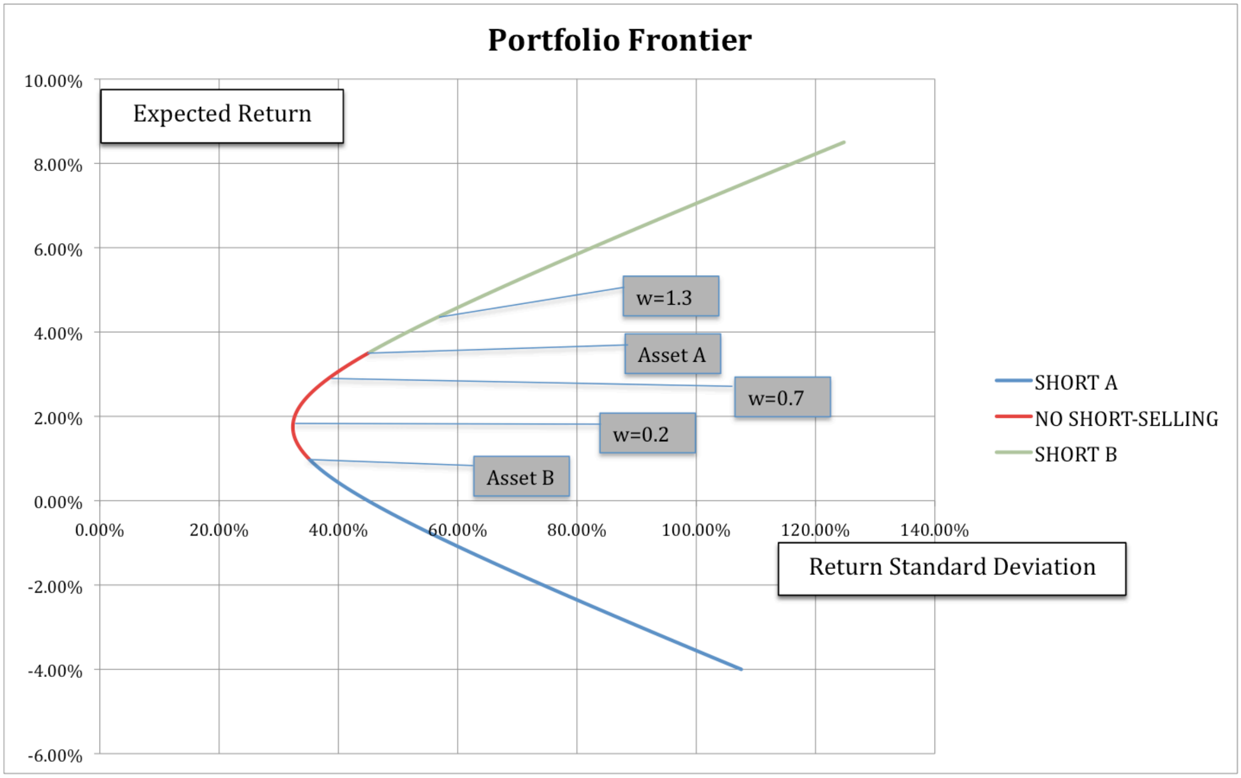
(b) As computed in part (a), the fair value of the described futures contract is $202. Therefore, when the actual futures price is $190, an investor can make a riskless profit without using any of his own capital with a "reverse cash and carry trading strategy":

- Period 0:
  1. Buy futures contract.
  2. Short-sell the underlying asset, and receive $200.
  3. Lend $200.
- Period 1:
  1. Receive $208=1.04•$200 from loan.
  2. Receive the asset from futures contract, and pay $190.
  3. Return the asset from futures contract, and pay $6 dividend to settle short-sale.

The total arbitrage profit from these transactions is $12.

Question 5

(a)


Portfolio Frontier
Expected Return
10.00%
8.00%
6.00%
4.00%
2.00%
0.00%
-2.00%
-4.00%
-6.00%
0.00%
20.00%
40.00%
60.00%
80.00%
100.00%
120.00%
140.00%
Return Standard Deviation
w=1.3
Asset A
w=0.7
w=0.2
Asset B
SHORT A
NO SHORT-SELLING
SHORT B

(b) The expected returns of all three portfolios do not depend on the correlation between assets A and B. Therefore, the expected returns are:

- w=1.3: 4.25% expected return,
- w=0.7: 2.75% expected return,
- w=0 (asset B): 1.0% expected return.

With respect to the return standard deviations, consider:

For w=1.3:

$$\begin{aligned} Var(r_P) &= Var(1.3\cdot r_A - 0.3\cdot r_B) \\ &= (1.3)^2\cdot Var(r_A) + (-0.3)^2\cdot Var(r_B) + 2\cdot 1.3\cdot(-0.3)\cdot Corr(r_A, r_B)\cdot Std(r_A)\cdot Std(r_B) \\ &= (1.3)^2\cdot(0.45)^2 + (-0.3)^2\cdot(0.35)^2 + 2\cdot 1.3\cdot(-0.3)\cdot(-0.4)\cdot 0.45\cdot 0.35 \approx 0.4024 = 40.24\%. \end{aligned}$$

$$Std(r_P) = \sqrt{Var(r_p)} = \sqrt{0.4024} \approx 0.6343 = 63.43\%.$$

For w=0.7:

$$\begin{aligned} Var(r_P) &= Var(0.7\cdot r_A + 0.3\cdot r_B) \\ &= (0.7)^2\cdot Var(r_A) + (0.3)^2\cdot Var(r_B) + 2\cdot 0.7\cdot 0.3\cdot Corr(r_A, r_B)\cdot Std(r_A)\cdot Std(r_B) \\ &= (0.7)^2\cdot(0.45)^2 + (0.3)^2\cdot(0.35)^2 + 2\cdot 0.7\cdot 0.3\cdot(-0.4)\cdot 0.45\cdot 0.35 \approx 0.0838 = 8.38\%. \end{aligned}$$

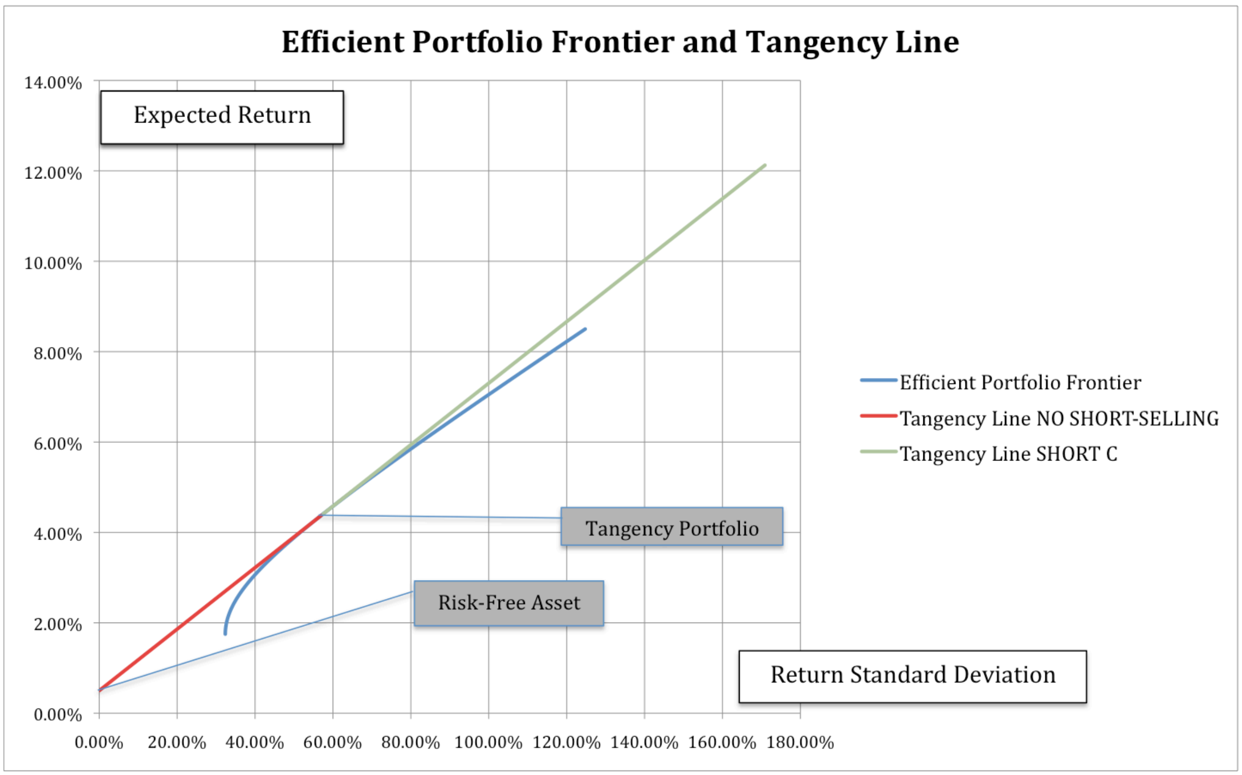
$$Std(r_P) = \sqrt{Var(r_p)} = \sqrt{0.086} \approx 0.2895 = 28.95\%.$$

w=0 corresponds to asset B. Its standard deviation is unaffected by changes the correlation between assets A and B. Therefore, the return standard deviation for w=0 is 35%.

(c) It is true that the decrease in correlation strictly decreases the return standard deviation for w=0.7 (and more generally for any 0<w<1) and it remains constant for w=0 (and more generally for w=0 and w=1). However, the return standard deviation strictly increases for w=1.3 (and more generally for w<0 and w>1). That is, in the latter cases, the described investor is not at least well off as before.

In consequence, lowering the correlation between assets A and B does not always entail diversification benefits.

(d)



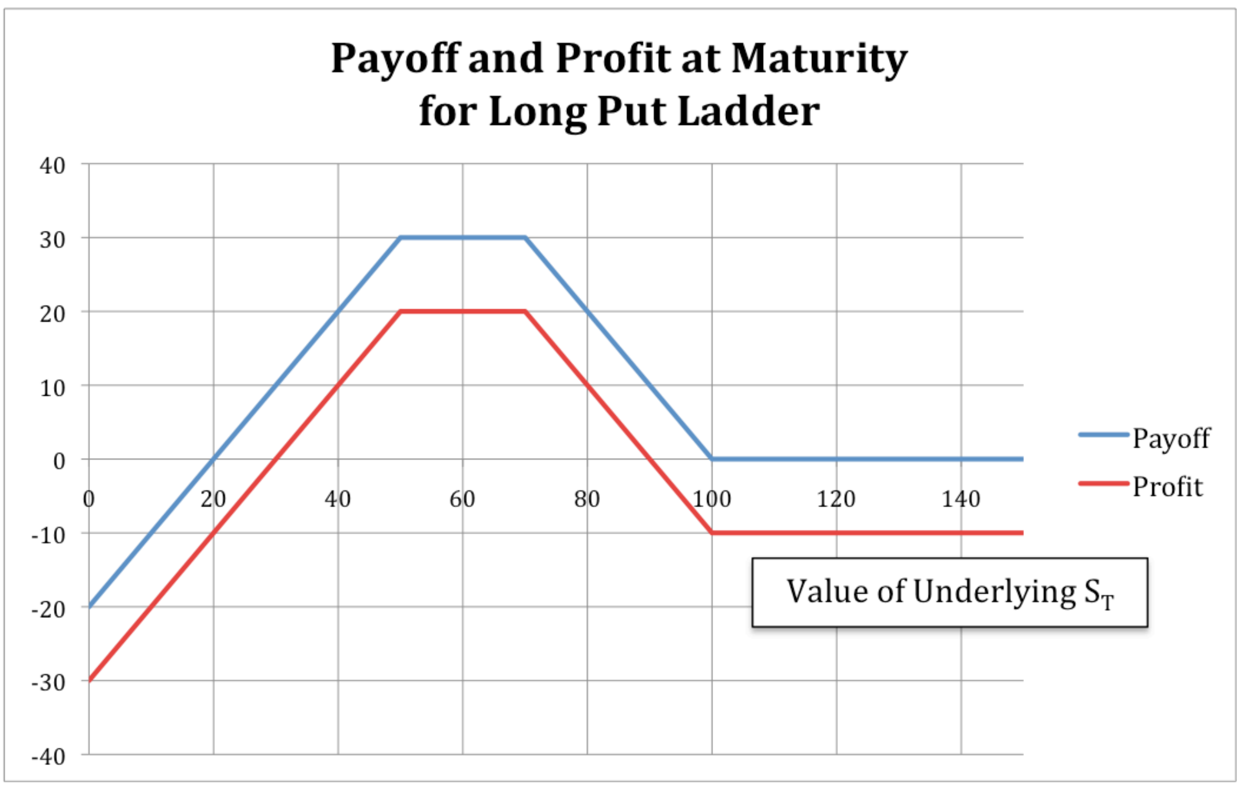
Question 6

The payoff of the described portfolio at maturity is obtained as follows:

Underlying	$S_T \leq 50$	$50 < S_T \leq 70$	$70 < S_T \leq 100$	$S_T > 100$
Payoff Put P_1 with E=100	$100 - S_T$	$100 - S_T$	$100 - S_T$	0
Payoff Put P_2 with E=70	$70 - S_T$	$70 - S_T$	0	0
Payoff Put P_3 with E=50	$50 - S_T$	0	0	0
Payoff Portfolio P_1-P_2-P_3	$S_T - 20$	30	$100 - S_T$	0

The portfolio P_1-P_2-P_3 costs \$55-\$29-\$16=\$10. It follows that the profit of the portfolio at maturity is as follows:

Underlying	$S_T \leq 50$	$50 < S_T \leq 70$	$70 < S_T \leq 100$	$S_T > 100$
Profit Portfolio P_1-P_2-P_3	$S_T - 30$	20	$90 - S_T$	-10



An investor might want to construct this portfolio if he thinks that the underlying security will experience little volatility between the time of construction and the maturity date. Moreover, he is more worried about price increases, which is why there is a floor in the losses from high prices, than he is worried about price decreases.

Question 7

The price of the underlying XYZ evolves as follows:

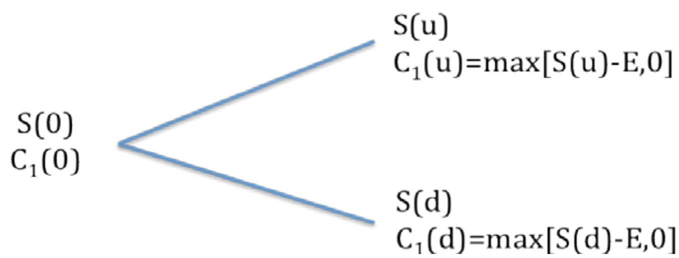
- A year from now:

  $$S(u) = u \cdot S(0) = 150, S(d) = d \cdot S(0) = 50.$$

- Two years from now:

  $$S(uu) = u^2 \cdot S(0) = 225, S(ud) = S(du) = u \cdot d \cdot S(0) = 75, S(dd) = d^2 \cdot S(0) = 25.$$

(a) The one-period Binomial Asset Pricing Model has the following schematic form:



The relevant quantity is the one-period hedge ratio for the call option C_1.

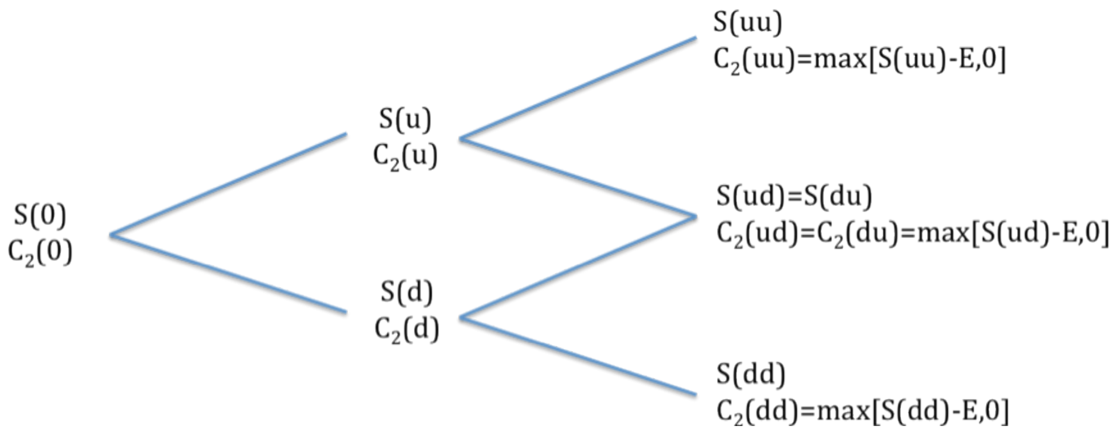
$$H = \frac{C_1(u) - C_1(d)}{(u-d) \cdot S(0)} \Leftrightarrow H = \frac{\max[150-60,0] - \max[50-60,0]}{(1.5-0.5) \cdot 100} = 0.9.$$

(b) The desired quantity is $C_1(0)$, which satisfies the following identity:

$$\frac{H \cdot S(u) - C_1(u)}{H \cdot S(0) - C_1(0)} = 1 + r \Leftrightarrow \frac{0.9 \cdot 150 - \max[150-60,0]}{0.9 \cdot 100 - C_1(0)} = 1.02$$

$$\Leftrightarrow C_1(0) \approx 45.88.$$

(c) The two-period Binomial Asset Pricing Model has the following schematic form:



The desired quantity $C_2(0)$ will be obtained via backward induction.

At the upper node in period 1, that is, after the stock price increases once:

$$H(u) = \frac{C_2(uu) - C_2(ud)}{(u-d)\cdot S(u)} \Leftrightarrow H(u) = \frac{\max[225-60,0] - \max[75-60,0]}{(1.5-0.5)\cdot 150} = 1.$$

Furthermore, the price of C_2 at the upper node in period 1 satisfies the following identity:

$$\frac{H\cdot S(uu) - C_2(uu)}{H\cdot S(u) - C_2(u)} = 1 + r \Leftrightarrow \frac{1\cdot 225 - \max[225-60,0]}{1\cdot 150 - C_2(u)} = 1.02$$
$$\Leftrightarrow C_2(u) \approx 91.18.$$

At the bottom node in period 1, that is, after the stock price increases once:

$$H(d) = \frac{C_2(du) - C_2(dd)}{(u-d)\cdot S(d)} \Leftrightarrow H(d) = \frac{\max[75-60,0] - \max[25-60,0]}{(1.5-0.5)\cdot 50} = 0.3.$$

Furthermore, the price of C_2 at the bottom node in period 1 satisfies the following identity:

$$\frac{H\cdot S(du) - C_2(du)}{H\cdot S(d) - C_2(d)} = 1 + r \Leftrightarrow \frac{0.3\cdot 75 - \max[75-60,0]}{0.3\cdot 50 - C_2(d)} = 1.02$$
$$\Leftrightarrow C_2(d) \approx 7.65.$$

Hence, $C_2(d)\approx\$7.65$ is the answer to the first question.

Finally, at the initial node:

$$H(0) = \frac{C_2(u) - C_2(d)}{(u-d)\cdot S(0)} \Leftrightarrow H(0) = \frac{91.18 - 7.65}{(1.5-0.5)\cdot 100} = 0.8353.$$

Furthermore, the price of C_2 at the initial node satisfies the following identity:

$$\frac{H\cdot S(u) - C_2(u)}{H\cdot S(0) - C_2(0)} = 1 + r \Leftrightarrow \frac{0.8353\cdot 150 - 91.18}{0.8353\cdot 100 - C_2(0)} = 1.02$$
$$\Leftrightarrow C_2(0) \approx 50.08.$$

Hence, $C_2(0)\approx\$50.08$ is the answer to the second question.

(d) Recall that put-call parity is

$$C+\frac{E}{(1+r)^T}=S+P,$$

where T denotes time to maturity. It holds at any time period and for any state of the stock price evolution.

Therefore, the price of a 2-year put with the same strike price as C_2 a year from now after the price has gone down once is

$$7.65+\frac{60}{1+0.02}=50+P(d)\Leftrightarrow P(d)=16.47.$$

Analogously, the price of this put option today is:

$$50.08+\frac{60}{(1+0.02)^2}=100+P(0)\Leftrightarrow P(0)=7.75.$$