Over the past 2 years, Prof. Bailyn has taken spectrographic observations of the star ‘Microscopium Ω’ using a powerful telescope in Chile. This is a small star (only $0.25M_\odot$), which has a prominent spectral line at a wavelength of 6000 Å. He thinks that there might be a planet orbiting this star, because the spectral line changes wavelength between observations. He’s too busy to fully analyze the data, however, and hands you the following chart (and some graph paper) with instructions to determine the mass of the planet orbiting this star.

The following equations might be helpful:

\[
\begin{align*}
  a^3 &= P^2M \\
  v &= 2\pi a/P \\
  \Delta \lambda/\lambda_0 &= v/c \\
  v_pM_* &= v_pM_p \\
  c &= 3 \times 10^8 \text{ m/s} \\
  1 \text{ AU} &= 1.5 \times 10^{11} \text{ m} \\
  1 \text{ yr} &= 3 \times 10^7 \text{ seconds}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Observation Date</th>
<th>Wavelength (Å)</th>
<th>Time (years)</th>
<th>$\Delta \lambda$</th>
<th>Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 2, 200.5</td>
<td>6000.0000</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Aug 31, 200.5</td>
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<td>-60</td>
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<td>May 2, 200.6</td>
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<td>2</td>
<td>+0.0012</td>
<td>+60</td>
</tr>
</tbody>
</table>

ANSWERS (note: star is assumed to have no systemic velocity)

\[
\begin{align*}
  a &= (P^2M)^{1/3} = (4*0.25)^{1/3} = 1 \text{ AU} \\
  V_r (\text{planet}) &= 2\pi a/P = 2\pi 1.5 \times 10^{11} / 2 \times 3 \times 10^7 = 15 \text{ km/s} \\
  M_p &= M_* (v_/v_p) = 2.5 \times 10^{-1} (6 \times 10^3 / 15 \times 10^3) = 15 / 15 \times 10^3 = 1 \times 10^3 \ M_\odot = 1 \text{ Jupiter Mass Planet!!}
\end{align*}
\]