1. The impedance of the circuit is given by

\[ Z(\omega) = R + \frac{1}{i\omega C} + i\omega L \]  

(1)

\[ = R + i(\omega L - \frac{1}{\omega C}). \]  

(2)

Noting the relation between the amplitudes, \(|I| = |V|/|Z|\), we have

\[ \frac{|I(\omega)|}{|I_{\text{max}}|} = \frac{|I(\omega)|}{|I(\omega_0)|} = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}. \]  

(3)

When \(\omega = \omega_0 \pm \delta = \omega_0 \pm R/2L\), provided \(\delta/\omega_0 < < 1\), we have

\[ \frac{|I(\omega_0 \pm \delta)|}{|I_{\text{max}}|} = \frac{R}{\sqrt{R^2 + \left((\omega_0 \pm \delta) L - \frac{1}{\omega_0 \pm \frac{\delta}{\omega_0}}\right)^2}} \]  

(4)

\[ = \frac{R}{\sqrt{R^2 + \left((\omega_0 \pm \delta) L - \frac{1}{\omega_0}(1 \mp \frac{\delta}{\omega_0})\right)^2}} \]  

(5)

\[ = \frac{R}{\sqrt{R^2 + (\pm \delta L \pm \frac{1}{\omega_0 \delta})^2}} \]  

(6)

\[ = \frac{R}{\sqrt{R^2 + \left(\frac{R}{2} \pm \frac{R}{2}\right)^2}} \]  

(7)

\[ = \frac{1}{\sqrt{2}}. \]  

(8)

2. From the relation \(1/Z_{\parallel} = \sum 1/Z_i\), we get

\[ \frac{1}{Z} = \frac{1}{R} + \frac{1}{i\omega C} + \frac{1}{i\omega L} \]  

(9)

\[ = \frac{1}{R} + \omega C - \frac{i}{\omega L}, \]  

(10)

and therefore,

\[ Z = \frac{1}{\frac{1}{R} + i\omega C - \frac{i}{\omega L}} = \frac{R\omega L}{\omega L + i(\omega^2 CL - 1)R}. \]  

(11)
3. The impedance of the circuit element shown in the figure satisfies the relation

\[
\frac{1}{Z} = \frac{1}{i\omega C} + \frac{1}{R + i\omega L} = i\omega C + \frac{1}{R - i\omega L} = i\omega C + \frac{R^2 + \omega^2 L^2}{R^2 + \omega^2 L^2} = R + i\left((R^2 + \omega^2 L^2)\omega C - \omega L\right). \tag{12}
\]

Noting that \(\text{Im}[Z]=0\) (\(Z\) is real.) \(\Leftrightarrow \text{Im}[1/Z]=0,\) we have

\[
\text{Im}[Z] = 0 \Leftrightarrow (R^2 + \omega^2 L^2)\omega C - \omega L = 0 \tag{13}
\]

\[
\Leftrightarrow (R^2 + \omega^2 L^2)C - L = 0, \text{ or } \omega = 0 \tag{14}
\]

\[
\Leftrightarrow \omega = 0, \sqrt{\frac{L - CR^2}{CL^2}}. \tag{15}
\]

Of course, \(\sqrt{\frac{L - CR^2}{CL^2}}\) is real only if \(L > CR^2.\) Otherwise, the impedance is real only for \(\omega = 0\) (Note that \(Z = \infty\) for \(\omega = 0\)).

4. As seen in problem 1, the impedance is given by

\[
Z(\omega) = R + \frac{1}{i\omega C} + i\omega L = R + i(\omega L - \frac{1}{\omega C}). \tag{20}
\]

Clearly, \(R_1 = 100 \, \Omega\) gives the minimum impedance, and \(R_2 = 200 \, \Omega\) gives the maximum impedance. Next, we have to consider the imaginary part of the impedance. For \(\omega = 2000,\) we get

\[
\omega L_1 - \frac{1}{\omega C_1} = 2000 \, s^{-1} \times 1 \, \text{mH} - \frac{1}{2000 \, s^{-1} \times 1 \, \mu F} = -498 \, \Omega, \tag{22}
\]

\[
\omega L_1 - \frac{1}{\omega C_2} = 2000 \, s^{-1} \times 1 \, \text{mH} - \frac{1}{2000 \, s^{-1} \times 100 \, \mu F} = -3 \, \Omega, \tag{23}
\]

\[
\omega L_2 - \frac{1}{\omega C_2} = 2000 \, s^{-1} \times 2 \, \text{mH} - \frac{1}{2000 \, s^{-1} \times 1 \, \mu F} = -496 \, \Omega. \tag{24}
\]
\( \omega L_2 - \frac{1}{\omega C_2} = 2000 \text{ s}^{-1} \times 2 \text{ mH} - \frac{1}{2000 \text{ s}^{-1} \times 100 \mu \text{F}} = -1 \Omega. \)  \hspace{1cm} (25)

Therefore, \((R_1, C_2, L_2)\) gives the minimum impedance \(|Z_{\text{min}}| = \sqrt{100^2 + 1^2} \approx 100 \Omega\), and \((R_2, C_1, L_1)\) gives the maximum impedance \(|Z_{\text{max}}| = \sqrt{200^2 + 498^2} \approx 537 \Omega\).

5. The impedance \(Z_2\) at \(\omega = 500\) is given by
\[
Z_2(\omega = 500) = 15 \Omega + \frac{1}{i \times 500 \text{ s}^{-1} \times 2 \mu \text{F}} = (15 - 1000i) \Omega \approx 1000.1 e^{-1.556i} \Omega,
\]
and the total impedance is
\[
Z_{\text{tot}}(\omega = 500) = (25 - 1000i) \Omega \approx 1000.3 e^{-1.545i} \Omega.
\]  \hspace{1cm} (26)

Using these, we can calculate the power loss across \(Z_2\). However, we have to note that \(P_2 = I_2 V_2 = \text{Re}[\hat{I}_2]\text{Re}[\hat{V}_2] \neq \text{Re}[\hat{I}_2\hat{V}_2]\), where \(\hat{A}\) is the imaginary expression of \(A\). (Operations such as derivative or integration commute with an operation of taking \(\text{Re}[\cdot]\), that is, the order of operations does not matter. Actually, this fact makes use of complex number convenient for this kind of problems. However, multiplication does not commute with \(\text{Re}[\cdot]\). Also note that complex numbers are "imaginary" tool to make calculation easier and that physical quantities we can observe in experiments are always real.) Therefore,

\[
P_2 = I_2 V_2 = I(Z_2) = \left( \frac{V}{Z_{\text{tot}}} \right) \frac{V Z_2}{Z_{\text{tot}}} = \text{Re}[\frac{30e^{j500t}[V]}{1000.3 e^{-1.545i} \Omega}] \text{Re}[\frac{30e^{j500t}[V](1000.1 e^{-1.556i} \Omega)}{1000.3 e^{-1.545i} \Omega}] = 0.900 \text{ Re}[e^{j(500t+1.545)}] \text{ Re}[e^{j(500t-0.011)}] [\text{W}] = 0.900 \cos(500t + 1.545) \cos(500t - 0.011) [\text{W}] = 0.450 \{ \cos(1000t + 1.534) + \cos 1.556 \} [\text{W}] \]

Also from this, we can easily calculate
\[
\text{Time average of power loss} = 0.450 \cos 1.556 = 6.66 \text{ mW}.
\]  \hspace{1cm} (34)
6. The electric field between the plates is

\[ E(r) = \begin{cases} \frac{V(t)}{d} & (r < a) \\ 0 & (r > a) \end{cases} \]  

(35)

where \( d = 2 \text{ cm} \) is the separation between the plates and \( a = 4 \text{ cm} \) is a radius of the plates. Noting that the capacitance has rotation symmetry about the central axis, we have from Maxwell equation,

\[
\oint \mathbf{B} \cdot d\mathbf{l} = 2\pi r B_\theta(r) = \epsilon_0 \mu_0 \int dS \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{c^2} \int dS \frac{\partial \mathbf{E}}{\partial t}.
\]

(36)

Therefore,

\[ B(r) = \begin{cases} \frac{r}{2\pi d} \frac{dV(t)}{dt} e_\theta & (r < a) \\ \frac{a^2}{2\pi c^2} \frac{dV(t)}{dt} e_\theta & (r > a) \end{cases} \]  

(37)

where \( \frac{dV(t)}{dt} = (-200\pi \times 200 \sin 200\pi t) \text{V/s} \), whose amplitude is \( 40000\pi \text{ V/s} \). \( B \) reaches its maximum at \( r = a \). With actual numbers plugged in,

\[
|B_{\text{max}}| = \frac{a}{2\pi c^2 d} \left| \frac{dV(t)}{dt} \right| = \frac{2\text{cm}}{2 \times (3 \times 10^8 \text{m/s})^2 \times 4\text{cm}} \times 40000\pi \text{V/s} = 1.11 \times 10^{-13} \text{T}.
\]

(39) (40)