I (5 points). Consider a binary star system consisting of two $1.5M_\odot$ neutron stars. Suppose one of the neutron stars is a pulsar with an average observed pulse period of exactly 2 seconds. If the orbit is circular and edge-on to our line of sight, what is the maximum and minimum pulse period observed? (Note: the pulse period $P_p$ obeys the same Doppler shift formulas as wavelength does).

II (5 points). Suppose a malevolent intelligence is trying to deceive us into thinking that a binary system of the kind described in problem 1 exists, when in fact that isn’t the case. Suppose this evil being lives on a neutron star that is not part of a binary system, and its scheme for deceiving us is to put a pulse-making machine on a movable platform that can go up and down. When the platform is high off the surface of the neutron star, the pulses suffer less gravitational redshift, and thus the observed pulse period is shorter — when the platform is on the surface of the neutron star, there will be more gravitational redshift, and $P_p$ will be longer. If the radius of the neutron star is $(4/3)R_s$, where $R_s$ is the Schwarzschild radius appropriate for the mass of the neutron star, it turns out that the difference in the observed pulse period due to the gravitational redshift between a high platform position and the surface is $\Delta P_p/P_p = 3h/R_n$, where $h$ is the height of the platform and $R_n$ is the radius of the neutron star. If the neutron star has mass $M_n = 1.5M_\odot$, how high does the platform have to be to reproduce the results of problem 1?

3 (3 points). The two situations in problems 1 and 2 could both provide a perfect fit to an observed sinusoidal velocity curve. But if such a velocity curve were observed from a pulsar, most people (and essentially all astronomers) would believe the existence of a binary system, rather than a malevolent intelligence. Why? Is this belief justified?

4 (6 points). The 1993 Nobel Prize for physics was awarded to Joe Taylor and Russell Hulse for the discovery of the first binary pulsar. Read Hulse’s entertaining account of their discovery (on the classes server resources page). What particular abilities and personal characteristics does Hulse have that enabled him to make this discovery? How do Hulse’s talents and discoveries compare with those of someone like Einstein? Do you think Hulse deserved the Nobel Prize, or did he just get lucky?

5 (1 point). Starting from the expression for the gravitational redshift (reproduced below) prove the result used in problem 2, that $\Delta P_p/P_p = 3h/R_n$. This problem is hard, and is worth only one point, so if you’re not fired up for some algebra, don’t bother.  Your starting point should be to take the gravitational redshift from the surface as viewed from infinity, and subtract the gravitational redshift from the top of the platform as viewed from infinity — that will give you the shift from the surface to the top of the platform. You can assume $h/R_n << 1$, but $R_s/R_n = 3/4$ in this case, and that is not a small quantity. That’s all the help you get with this one — if you can’t get it, don’t worry, it’s only one point. The relevant formula is

$$\Delta P_p/P_p = \frac{1}{\sqrt{1 - R_s/R}} - 1$$