Lecture 22  28 Nov 07

last time repeated interaction

need: gain if cheat

today [value of relationship after cooperation] - [value of relationship after cheating]

tomorrow

promise threat

Credibility: focus of SPE

Prisoner's dilemma repeated with prob δ of continuing

\[
\begin{array}{c|c|c|c}
\text{C} & \text{D} \\
\hline
\text{C} & 2,2 & -1,3 \\
\text{D} & 3,-1 & 0,0 \\
\end{array}
\]

grim trigger: play C, then

\[
\text{play } \begin{cases} \text{C if no one has ever defected} \\ \text{D otherwise} \end{cases}
\]

Temptation \[
\frac{3-2}{\delta}
\]

\[
2(\delta^2) + \delta^3 + \delta^4 + \ldots = \frac{\delta^2}{1-\delta}
\]

\[
\frac{\delta^3}{1-\delta} + \delta^4 + \delta^5 + \ldots = \delta X
\]

\[
X = 2 \frac{\delta}{1-\delta}
\]

<< Is grim trigger an equilibrium [when both play it]? >>

\[
\begin{align*}
1 & \leq \left[ \frac{2}{1-\delta} - 0 \right] \delta \\
\Leftrightarrow 1-\delta & \leq 2 \delta \\
\Leftrightarrow \delta & \geq \frac{1}{3}
\end{align*}
\]

*How about playing D now, then C, then D forever?*

\[
(D,C),(C,D),(D,D),(D,D) \rightarrow 3 + 3(1) + 0 + 0 \ldots = 3 - 8
\]

This defection is even worse (than the previous defection of D,D,D, ...)

Punishment (D,D) forever is a SPE

*How about cheating, not in the first period, but in the second?*

The same analysis says this is not profitable if \[ \delta \geq \frac{1}{3} \]

Lesson: we can get cooperation in PD (prisoners' dilemma) using grim trigger (as a SPE) provided \[ \delta \geq \frac{1}{3} \]

Lesson: For an ongoing relationship to provide incentives for good behavior, it helps for there to be a high probability that the relationship will continue. weight you put on the future

<< What about a less draconian strategy? >>

One-period punishment ...

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one period punishment

\[
\text{play } C \text{ to start, then}
\]
\[
\begin{align*}
\text{play } C \text{ if either } (C,C) \text{ or } (D,D) \text{ were played last} \\
\text{D if either } (C,D) \text{ or } (D,C) \text{ were played last}
\end{align*}
\]

is this an SPE?

\[
\text{temptation } \leq \left( \text{value of promise } \right) - \left( \text{value of threat } \right)
\]

\[
3 - 2 \leq \left( \frac{2}{1-\delta} \right) - \delta \left( \frac{2}{1-\delta} \right)
\]

\[
\Rightarrow \quad 1 \leq \frac{2\delta}{1-\delta} [1-\delta]
\]

\[
\Rightarrow \quad \frac{1}{2} \leq \delta
\]

Trade off
shorter punishments need more weight (\(\delta\)) on future

<< Example to show repeated interaction works >>
Repeated Moral Hazard

+ labor cheap
  - contracts hard to enforce

\[
\begin{array}{c}
\text{Invest } t \text{ (w)} \\
\text{Honest } 3 - w, w
\end{array}
\]

\[
\begin{array}{c}
\text{Cheat } -1, 2
\end{array}
\]

\[
\begin{array}{c}
\text{worse } 0, 1
\end{array}
\]

if set \(w = 1\) (the going wage in Fredonia)
then the agent will cheat
to make him be honest,
need \(w > 2\)
incentive design
In equilibrium, \(w^* = 2\), the agent works
Wage premium in this emerging market is 100%

\begin{itemize}
  \item Consider repeated interaction with prob \(\delta\)
  \item what wage \((w^*)\) will you pay?
  \item temptation to cheat today \(\leq \delta \left[ \left( \text{value of continuing the relationship } \right) - \left( \text{value of ending the relationship } \right) \right]
  \item "Continuing" "firing"
  \item \(2 - w^* \leq \left[ \left( \frac{w^*}{1-\delta} \right) \text{ forever } \right] - \left( \frac{1}{1-\delta} \right)\delta
  \item \(\left(1-\delta\right)^2 - \left(1-\delta\right) \text{ w}^* \leq w^* \delta - [1] \delta
  \item \(\left(1-\delta\right)^2 \delta [1] \leq w^*
\end{itemize}

\[
\text{or: } 2 - \delta \leq w^*
\]

\begin{itemize}
  \item if \(\delta = 0\), \(w^* = 2\) one-shot wage
  \item if \(\delta = 1\), \(w^* = 1\) going wage
  \item if \(\delta = \frac{1}{2}\), \(w^* = \frac{1}{2}\) wage premium is now only 50%
\end{itemize}

<< to get good behavior, must be a reward >>
<< size of reward related to prob. of future >>