Last time apply SPE  
- solve NE in each subgame  
- roll back payoffs

Lesson  
Strategic effects matter!  
- investment game  
- tax design  
- tolls

2 players  
each period each chooses  
For Q game ends as soon as someone Q's

Good news  
if the other player quits first,  
you win a prize  
\( V = \$1 \)

Bad news:  
each period in which both F  
each player pay cost  
\(-C = .75 \$\)

If both quit at once  
\( \rightarrow 0 \)

Examples  
- WWII  
- B2B v. Sky  
- Wars of Attrition  
- bribe contests

Two period game

Two cases:  
\( V > C \) \( \leftarrow \) here in class

\( V < C \) \( \leftarrow \) on homework

Second subgame

Two pure-strategy NE in this subgame:  
\( (F(1), q(1)) \), \( (Q(1), f(1)) \)

Payoffs  
\( (V, 0) \)

\( (0, V) \)

First stage revisited

Continuation payoffs

NE

\( (F(1), q(1)) \)

\( (V, 0) \)

\( (0, V) \)

\( (0, 0) \)
Pure strategy SPE (with \( v > 2c \))

\[
\begin{bmatrix}
(F(1), F(2)) & (q(1), q(2)) \\
(Q(1), Q(2)) & (f(1), f(2))
\end{bmatrix}
\]

"quitier v. fighter"

<< Now look for mixed strategy eq. >>

Second subgame

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(1)</td>
<td>C-C</td>
</tr>
<tr>
<td>F(2)</td>
<td>V, 0</td>
</tr>
<tr>
<td>Q(1)</td>
<td>0, V</td>
</tr>
<tr>
<td>Q(2)</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

\( p \) (1-p)

```
\text{Mixed NE in this matrix is: both } F \text{ with prob } p^* = \frac{\sqrt{v}}{V + c}
```

```
\text{Mixed SPE } \begin{bmatrix}
(p^*, p^*) & (p^*, p^*) \\
(p^*, p^*) & (p^*, p^*)
\end{bmatrix}
```

E payoff is 0

<< Not pride, craziness >> / in \( V \), \( v \) in \( C \)

Infinite period game

```
\text{Mixed NE has both fight with prob } \frac{v}{V + c}
```

payoffs in this mixed NE = (0, 0)

<< back to first stage >>

```
\text{Stage 2 NE payoffs: } -C + \text{Stage 2 NE payoffs}
```

For the mixed NE in period 2

```
\text{Stage 2 NE payoffs: } -C + \text{Stage 2 NE payoffs}
```

\( p \) (1-p)

\( \text{V+} \text{ continuation values} \)

\( \text{C+} \text{ continuation values} \)

<< Now this analysis is already solved! >>

Same conclusion, too:

both mix with prob \( F = p^* = \frac{\sqrt{v}}{V + c} \)