Zeimelo Theorem

- 2 players
- perfect information
- finite nodes
- three outcomes  \( W, L, T \)

Either 1 can force a win (for 1) or 1 can force a tie or 2 can force a loss (on 1)

- e.g. Nim unequal \( \rightarrow 1 \) can force a \( W \)
- equal \( \rightarrow 2 \) can force a \( L \)

- e.g. T.T.T. \( \rightarrow \) tie
- e.g. Chess

Proof (by induction) on maximum length of game \( N \)

- if \( N = 1 \)
  - \( W \)
  - \( T \)
  - \( L \)

Example \( N = 3 \)

\( N+1 = 4 \)

Induction hypothesis

By induction hypothesis, upper subgame has a solution.

Say \( \{W_i\} \)

By induction hypothesis, lower subgame has a solution.

Say \( \{L_i\} \)

So translate the above game to:

This has a solution, it is a game of length 1.

Claim: we're done with proof (by induction).

1. solution \( \checkmark \)
2. initial step, the game of 1. solution \( \checkmark \)
3. else 2 has solution, \( \Rightarrow \) 3 solution. \( \checkmark \)
Zeimelo's Thm: this game has a solution
(which could depend on \( N \times M \))

Homework: what's the solution?

Formal Stuff

Define: A game of perfect information is one in which at each node, the player whose turn it is to move knows which node she is at (and how she got there).

Define: A pure strategy for player \( i \) in a game of perfect information is a complete plan of actions; it specifies which action \( i \) will take at each of its decision nodes.

Example:

\[ \begin{array}{c}
\text{Entrant} \\
\text{Out}
\end{array} \]

\[ \begin{array}{c|c|c}
\text{IN} & -1,0 & 1,1 \\
\hline
\text{Out} & 0,3 & 0,3 \\
\end{array} \]

\[ \text{NE} = (\text{IN, NF}) \]

?? What is happening with this equilibrium?

It is a NE but relies on believing an incredible threat.

\[ \begin{array}{c|c|c}
\text{U} & 2,4 & 0,2 \\
\hline
\text{D} & 3,1 & 0,2 \\
\end{array} \]

\[ \begin{array}{c|c|c}
\text{Du} & 1,0 & 1,0 \\
\hline
\text{Dd} & 1,0 & 1,0 \\
\end{array} \]

\[ \text{NE} = ([Dd], r), ([Du], r) \]

\[ \text{BI} : ([Dd], r) \]

\[ \text{Equilibrium} \]

\[ \text{Found by BI} \]