Lecture 12  15 Oct 07

Defn In a 2-player symmetric game,
a strategy \( \hat{s} \) in ES (in pure strategies) if:
(i) \( (\hat{s}, \hat{s}) \) is a symmetric NE, AND
(ii) if \( \hat{s} \neq s \) is not strict NE,
then \( u(\hat{s}, \hat{s}) > u(s, s) \)

```
\[
\begin{array}{cc}
    a & b \\
    a & 1,1 & 1,1 \\
    b & 1,1 & 0,0 \\
\end{array}
\]
```

« What is Nash? » \((a, a)\) is sym. Nash
Is \((a, a)\) strict NE? \(\text{No: } u(a, a) = u(b, a) = 1\)
So check \(a > b\)
\[a + b > 0 \checkmark\]
\(\text{So } a \text{ is ES}\)

Evolution of social convention: driving on Loc R

```
\[
\begin{array}{cc}
    L & R \\
    L & 2,2 & 0,0 \\
    R & 0,0 & 1,1 \\
\end{array}
\]
```

« What are potential ES? »
\((L, L)\) are both NE
\((R, R)\) are both NE
Strict, so \(L\) is ES
\(R\) is ES

Lesson: We can have multiple ES conventions.
These need not be equally good.
« (2,2) "Better than" (1,1) »

\[\begin{array}{cccc}
    & a & b \\
    a & 0,0 & 2,1 \\
    b & 1,2 & 0,0 \\
\end{array}\]

\[\text{Nature interpretation: }\begin{array}{cc}
a & \text{aggression} \\
b & \text{non-aggression} \\
\end{array}\]

There is no symmetric pure-strategy NE in the game.
\[\left(\frac{2}{3}, \frac{1}{3}\right) \text{ is NE (polymorphic).}\]

Defn change:
\[\hat{s} \rightarrow \hat{p}\]
pure \(\rightarrow\) mixed

\[\text{mixed eq. cannot be strict, since it is mixed}\]
\[\text{need to check } u(\hat{p}, \hat{p}) > u(p, p') \text{ for all possible mixed mutations } p'.\]

Hawk-Dove (strategy names for same species)

```
\[
\begin{array}{cccc}
    & H & D \\
    H & \frac{V-C}{2} & \frac{V}{2} \\
    D & \frac{V}{2} & C \\
\end{array}
\]
```

\[\text{prize } V > 0\]
\[\text{costs } C > 0\]

\[\text{(Is } D \text{ on } \text{ESS?)}\]
\((D, D)\) a NE? \(\times\) so not ESS
\((H, H)\) a NE? \(\text{Yes if } \frac{V-C}{2} > 0\)

Case 1: \(V > C\)
then \((H, H)\) is strict NE
\(\text{Case 1: } V = C \Rightarrow \frac{V-C}{2} = 0\)
\(u(H, H) = u(D, D)\)
Check \( U(H, D) \geq U(D, D) \)
\[
\forall > \frac{V}{2} \quad \checkmark
\]
\( )

Shown: if \( \forall > C \)
the \( H \) is an ESS

If \( C > V \)
we know \( H \) is not ESS
\( D \) is not ESS

What about \( p^* \)?

Step one: find a symmetric mixed NE \( \{ p^*, 1-p^* \} \)
\[
\begin{align*}
U(H, p^*) &= p^* \left( \frac{V+C}{2} \right) + (1-p^*) V \\
U(D, p^*) &= \left( \frac{C}{2} \right) + (1-p^*) V
\end{align*}
\]
\[
\left\{ \begin{array}{c}
\hat{p} = \frac{V}{C} \\
\left( \frac{V}{C}, 1-\frac{V}{C} \right)
\end{array} \right\
\]

The only hope for an ESS is \( \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) \)

\( \left\{ \begin{array}{c}
\text{It is NE,} \\
\text{It is weak NE}
\end{array} \right\}
\]

Check \( U(p^*, p') \geq U(p_0', p') \)

Let \( p' = S \)
\[
U(p^*, S) = \frac{1+V}{3} < 1
\]

\( U(S, S) = 1 \quad \checkmark \) bigger.

Example: \( \boxed{\text{NO ESS}} \)

What happens?

Cycling around!
orange lizards - hares
yellow lizards - sneaky
blue lizards - monogamy

Lessons:
If \( V < C \), then ESS has \( \frac{V}{c} \) Hawks
a) as \( V \uparrow \), more Hawks in ESS
as \( C \uparrow \), more Doves in ESS
b) payoffs = \( (1-\frac{V}{C}) \frac{V}{2} \)
\( \left\{ \begin{array}{c}
\text{Dove payoff,} \\
\text{must be same as Hawk payoff}
\end{array} \right\}
\)
what happens as \( C \uparrow ? \) the payoff \( \uparrow \)
c) identification we can tell what \( \frac{V}{C} \) is
from data