Penalty Kick Game
Portsmouth v. Liverpool

\[
\begin{array}{c|c|c}
\text{goalie} & \text{L} & \text{R} \\
\text{L} & 4,4 & 1,9 \\
\text{M} & 6,6 & 6,6 \\
\text{R} & 9,9 & 4,4 \\
\end{array}
\]

\[U_i(L, L) = 4, \text{ i.e. 40\% chance of scoring}\]

\[EU_i(L, p(r)) \quad EU_i(M, p(r)) \quad EU_i(R, p(r))\]

\[\text{Belief } p(r)\]

\[\text{M is not a BR to any belief}\]

**Lesson**: Do not shoot to middle (unless you are German)

**Lesson**: Do not choose a strategy that is never a BR to any belief

- "L" "R"
- "L" 63.6 94.4 "L" = natural
- "R" 89.3 43.7

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**Partnership Game**

- 2 agents own firm jointly, share 50\% of profit each
- Each agent chooses effort level to put into the firm
  \[s_i^c = [0, 1]\] "continuum of strategies"
- Firm profit is given by

\[4[s_1 + s_2 + b(s_1 s_2)]\]

Complementarity / Synergy

\[0 \leq b \leq \frac{1}{4}\]

- Payoffs
  \[U_1(s_1, s_2) = \frac{1}{2} [4(s_1 s_2 + b s_1 s_2)] - s_2^2\]
  \[U_2(s_1, s_2) = \frac{1}{2} [4(s_1 s_2 + b s_1 s_2)] - s_2^2 + \text{cost}\]

\[\max_{s_i} 2(s_1 + s_2 + b s_1 s_2) - s_i^2\]
Differentiate

\[ \text{f.o.c.} \quad 2 \left(1 + bs_2^\prime\right) - 2s_1^\prime = 0 \]

\[ \text{s.o.c.} \quad -2 < 0 \checkmark \]

\[ s_1^\prime = 1 + bs_2 = \text{BR}_1(s_2) \]

\[ s_2^\prime = 1 + bs_1 = \text{BR}_2(s_1) \]

Similarly,

\[ s_2 = 1 + bs \]

\[ s_1 = 1 + bs \]

\[ \text{Nash Equilibrium} \quad \langle \text{intersection of lines} \rangle \]

\[ \text{The players are playing a best response to each other} \]

\[ \text{draw } \text{BR}_1, \text{BR}_2 \text{ for the case } b = \frac{1}{4} \]

\[ \text{BR}_1(s_2) = 1 + \frac{1}{4}s_2 \]

\[ s_1^* = 1 + bs_2^* \quad \rightarrow \quad s_1^* = s_2^* \]

\[ (1 + b)s_1^* = 1 \quad \rightarrow \quad s_1^* = s_2^* = \frac{1}{1 - b} \]