ASTRO 160
(part deux)
BLACK HOLES & RELATIVITY
(note: no problem set this week)
BLACK HOLES
not seen directly
use - orbital dynamics, Doppler shift etc.
content: relativity not Newtonian physics

"escape velocity"

\[ V_{esc} = \left( \frac{2GM}{R} \right)^{1/2} \]

speed required to escape grav. field of an object with mass \( M \) radius \( R \)
(if you are on the surface)
V = V_\text{esc} \quad L \text{couter to } 00

V_{\text{esc}} = \left(\frac{2GM}{R}\right)^{\frac{1}{2}}

V < V_{\text{esc}}

V > V_{\text{esc}}

V < V_{\text{esc}}

V_{\text{esc}} \text{ of Earth}

V_{\text{esc}} = \left(\frac{2GM}{R}\right)^{\frac{1}{2}} = \left(\frac{2 \times 7.2 \times 10^{-11} \text{ m} \text{ kg}^{-2} \text{ s}^{-2} \times 6 \times 10^{24} \text{ kg}}{7 \times 10^6 \text{ m}}\right)^{\frac{1}{2}}

= \left(12 \times 10^8 \text{ m}^2 \text{ s}^{-2} \text{ kg}^{-1}\right)^{\frac{1}{2}} \approx \left(1.21 \times 10^8 \text{ m}^2 \text{ s}^{-2} \text{ kg}^{-1}\right)^{\frac{1}{2}}

\approx 10^7 \text{ m/s}

\approx 10 \text{ km/s}
\[ \text{Vesc of human} \]

\[ M = 100 \text{ kg} \]
\[ R = 1 \text{ m} \]

\[ \text{Vesc} = \left( \frac{2GM}{R} \right)^{\frac{1}{2}} \]

\[ = \left( \frac{2 \times 7 \times 10^{-11} \times 10^2}{1} \right)^{\frac{1}{2}} \]

\[ = (1.4 \times 10^{-8})^{\frac{1}{2}} \]

\[ = 1 \times 10^{-4} \text{ m/s} \]

\[ < 1 \text{ m/hr} \]

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**Black Hole:** \[ \text{Vesc} \geq 0.07 \times 10^5 \text{ m/s} \]
\[ V_{esc} = c \]
\[ c = \left( \frac{2GM}{R} \right)^{1/2} \]
\[ R_s = \frac{2GM}{c^2} \]

Schwarzschild radius

black hole:
\[ R < R_s \]

FABLE: Mitchell's "discovery" of black holes

MORAL: importance of result changes with context
how big is Rs of Sun?

\[ Rs = \frac{2GM}{c^2} = \frac{2 \times 7 \times 10^5 \text{ m s}^{-2}}{(3 \times 10^8 \text{ m s}^{-1})^2} \]

\[ = \frac{3 \times 10^{10}}{10^6} = \frac{3 \times 10^3 \text{ m}}{10^6} = 3 \text{ km} \]

Known for 3,700 yrs.

Mat black holes should exist - endpoints of stellar evolution.

Star's evolution determined by two forces

- gravity (pulls in)
- pressure (push out)

"hydrostatic equilibrium"
$P_{\text{in}} \rightarrow P_{\text{out}}$ due to gravity

$P_{\text{in}} - P_{\text{out}}$ = Gravity

$P_{\text{in}} > P_{\text{out}}$

"gas pressure" $\rightarrow$ const.

$PV = nRT$

$\frac{\n}{V} = g \times \text{const.}$

$P = \text{const} \times g \times T$
inside T, S bigger than outside

if only S varies
inner regions $\Rightarrow$ higher pressure
$\Rightarrow$ higher gravity

no balance is possible

$\Rightarrow$ inner parts of star
must be hotter than outer part

inside of Sun $10^7$ degree
surface of Sun $6 \times 10^3$ degree

heat in center of star flows out (at surface radiation)
require: an energy source at center of star

→ replaces lost heat
→ preserves equilibrium

NUCLEAR FUSION

eventually nuclear fuel runs out

→ many advantages

→ collapses

at high density

→ other kinds of pressure

"electron degeneracy pressure" (Fermi pressure)

stabilizes start of arend radius of Earth
\[ S = \frac{M}{\sqrt{\frac{2 \times 10^{30}}{4 \times 350 \times 10^{18}}} \left( \frac{2 \times 10^{33}}{1400 \times 10^{18}} \right)} = 10^{9} \text{m/}^2 \text{m} \]

Million times denser than water

Such stars called white dwarf end part of Sun

1930s Chandrasekhar proves that
\[ M > 1.4 \text{M}_\odot \]

At electron pressure insufficient
FABLE: Chandra's Limit
MORAL: believe your shoot not your intuition

When white dwarf collapses
\[ e + p \rightarrow n + \nu \rightarrow \text{neutrons} \]
whole star turns into neutrons

NEUTRON STARS
(discovered in 1960s)

\[ M = 2.4 M_\odot \quad R = 10 \text{km} \]

\[ S \text{ is a billion times greater than white dwarf!} \]

\[ \frac{R_{\odot}}{R} = \frac{2GM}{c^2} \approx \frac{2GM}{c^2} \text{ for low mass} \]
If $R_{so} = 3 \text{ km}$

\[ R_{s+} = 10 \text{ km} \]

with $M = 3M_\odot$

$= \text{radius of neutron star}$

neutron star with $M = 3M_\odot$

\[ R < R_{s+} \rightarrow \text{black hole} \]

shrs with initial mass $> 30M_\odot$

will end up with $M > 3M_\odot$

$\rightarrow$ collapse $\rightarrow$ black hole

$\Rightarrow$ \text{expect there are many black holes}