Lecture 10: Debt Markets and Term Structure

Economics 252, Spring 2008
Prof. Robert Shiller, Yale University
## Historical Securities Search Results

### Treasury Bills

**Auction Dates: From:** Feb 8, 2008  **To:** Feb 15, 2008  
**Security Terms:** All  
**Sorted By:** Auction Date in Descending order

<table>
<thead>
<tr>
<th>Security Term</th>
<th>Auction Date</th>
<th>Issue Date</th>
<th>Maturity Date</th>
<th>Discount Rate %</th>
<th>Investment Rate %</th>
<th>Price Per $100</th>
<th>CUSIP</th>
</tr>
</thead>
<tbody>
<tr>
<td>63-DAY</td>
<td>02-13-2008</td>
<td>02-14-2008</td>
<td>04-17-2008</td>
<td>2.440</td>
<td>2.491</td>
<td>99.570000</td>
<td>912795D81</td>
</tr>
<tr>
<td>4-WEEK</td>
<td>02-12-2008</td>
<td>02-14-2008</td>
<td>03-13-2008</td>
<td>2.500</td>
<td>2.547</td>
<td>99.805556</td>
<td>912795D32</td>
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<tr>
<td>26-WEEK</td>
<td>02-11-2008</td>
<td>02-14-2008</td>
<td>08-14-2008</td>
<td>2.080</td>
<td>2.137</td>
<td>98.948444</td>
<td>912795F97</td>
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<tr>
<td>13-WEEK</td>
<td>02-11-2008</td>
<td>02-14-2008</td>
<td>05-15-2008</td>
<td>2.250</td>
<td>2.301</td>
<td>99.431250</td>
<td>912795E49</td>
</tr>
</tbody>
</table>
Discount Bonds Pricing

Term $T$, Yield to Maturity (YTM) $r$

$$P_t = \frac{1}{(1+r)^T}$$

$$P_t = \frac{1}{(1+r/2)^{2T}}$$
Compound Interest

• If annual rate is $r$, compounding once per year, balance = $(1+r)^t$ after $t$ years.
• If compounded twice per year, balance is $(1+r/2)^{2t}$ after $t$ years.
• If compounded $n$ times per year, balance is $(1+r/n)^{nt}$ after $t$ years.
• Continuous compounding, balance is $e^{rt}$. 
Price & Yield on T-Bills

- For buyer, Price = 100 - Discount
- Discount = Asked*(Days to Maturity/360). (Same as formula on page 295 of Fabozzi, where D=Discount, F=100, Y=Asked/100, t=Days)
- Yield = (Discount/Price)(365/(Days to Maturity)). (Unless maturity > 6 months, in which case quadratic formula using semi-annual compounding is required.)
Conventional Bonds Carry Coupons

- Conventional Bond Issued at par (100), coupons every six months.
- Term is time to maturity.

\[
P_t = c\left(\frac{1}{r} - \frac{1}{(1+r)^T} \frac{1}{r}\right) + \frac{100}{(1+r)^T}
\]

\[
P_t = \frac{c}{2} \left(\frac{1}{r/2} - \frac{1}{(1+r/2)^{2T}} \frac{1}{r/2}\right) + \frac{100}{(1+r/2)^{2T}}
\]
### Historical Securities Search Results

**Treasury Notes**

**Auction Dates:** From: Jan 1, 2008  To: Feb 15, 2008  
**Security Terms:** All  
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<table>
<thead>
<tr>
<th>Security Term</th>
<th>Auction Date</th>
<th>Issue Date</th>
<th>Maturity Date</th>
<th>Interest Rate %</th>
<th>Yield %</th>
<th>Price Per $100</th>
<th>CUSIP</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-YEAR</td>
<td>01-29-2008</td>
<td>01-31-2008</td>
<td>01-31-2013</td>
<td>2.875</td>
<td>2.909</td>
<td>99.842844</td>
<td>912828HQ6</td>
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<td>2-YEAR</td>
<td>01-28-2008</td>
<td>01-31-2008</td>
<td>01-31-2010</td>
<td>2.125</td>
<td>2.237</td>
<td>99.782126</td>
<td>912828HP8</td>
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</tbody>
</table>
# Historical Securities Search Results

**Treasury Bonds**

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**Security Terms:** All  
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<tr>
<td>20-YEAR</td>
<td>01-24-2008</td>
<td>01-31-2008</td>
<td>01-15-2028</td>
<td>1.750</td>
<td>1.807</td>
<td>99.351033</td>
<td>912810PV4</td>
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</tbody>
</table>
Forward Rates

J. R. Hicks *Value and Capital* 1939

\[
(1 + r_2)^2 = (1 + r_1)(1 + f_2)
\]

\[
(1 + r_k)^k = (1 + r_{k-1})^{k-1}(1 + f_k)
\]
Jan 2008 Term Structure

Percent Yield vs. Years to Maturity
Dec 2003 Term Structure

Percent Yield

0.0  0.5  1.0  1.5  2.0  2.5  3.0  3.5  4.0

Years to Maturity

0  1  2  3  4  5  6  7  8
Inflation and Interest Rates

• Nominal rate quoted in dollars, real rate quoted market baskets
• Nominal rate usually greater than real rate.

\[
(1 + r_{money}) = (1 + r_{real})(1 + i)
\]

\[
r_{money} \equiv r_{real} + i
\]
STATE of MASSACHUSETTS BAY

The First Day of January, A. D. 1783;

InBehalf of the State of Massachusetts Bay, I the Subscriber do hereby promise and oblige Myself and Successors in the Office of TREASURER of said State, to pay unto John Peper or to his Order, the Sum of Four hundred and fifty-four pounds. to pay unto John Peper or to his Order, the Sum of Four hundred and fifty-four pounds.

On or before the First Day of March, in the Year of our Lord One Thousand Seven Hundred and Eighty-four, with Interest at Six per Cents per Annum: Both Principal and Interest to be paid in the then current Money of said State, in a greater or less Sum, according annum:

Five Bushels of CORN, Sixty-eight Pounds and Four-seventh Parts of a Pound of BEEF, Ten Pounds of SHEEP WOOL, and Sixteen Pounds of SOLE LEATHER shall then cost, more or less than One Hundred and Thirty Pounds current Money, at the then current Prices of said Articles. This Sum being Thirty Two Times and an Half what the same Quantities of the same Articles would cost at the Prices ascribed to them in a Law of this State made in the Year of our Lord One Thousand Seven Hundred and Seventy-seven, intituled, "An Act to prevent Monopoly and Oppression." The current Prices of said Articles, and the consequent Value of every Pound of the Sum herein promised, to be determined agreeable to a Law of this State, intitled, "An Act to provide for the Security and Payment of the Balances that may appear to be due by Virtue of a Resolution of the General Assembly of the Sixth of February One Thousand Seven Hundred and Seventy-nine, to this State's Quota of the CONTINENTAL ARMY, agreeable to the Recommendation of CONGRESS, and for Supplying the TREASURY with a Sum of Money for that Purposes."

No John Peper

Witness my Hand.

H. Channing, Treasurer.