Optimal Portfolio Diversification in General Case

• Drop assumption of equal weighting, independence and equal variance
• Put $x_i$ dollars in $i$th asset, $I=1,..,n$, where the $x_i$ sum to $1$.
• Portfolio expected value
  $$r = \sum_{i=1}^{n} x_i E(\text{return}_i) = \sum_{i=1}^{n} x_i r_i$$
• Portfolio variance (two assets) =
  $$x_1^2 \text{var}(\text{return}_1) + (1-x_1)^2 \text{var}(\text{return}_2) + 2x_1(1-x_1)\text{cov}(\text{return}_1, \text{return}_2)$$
Efficient Portfolio Frontier with Two Assets

• Frontier expresses portfolio standard deviation in terms of portfolio expected return $r$ rather than in terms of $x_1$.

• 

\[ x_1 = \frac{r - r_2}{r_1 - r_2} \]

\[ \sigma^2 = \left( \frac{r - r_2}{r_1 - r_2} \right)^2 \sigma_1^2 + \left( \frac{r_1 - r}{r_1 - r_2} \right)^2 \sigma_2^2 + 2 \frac{(r - r_2)(r_1 - r)}{(r_1 - r_2)^2} \sigma_{12} \]
Portfolio Variance, Three Assets

- Portfolio variance =

\[ x_1^2 \text{var}(return_1) + x_2^2 \text{var}(return_2) + x_3^2 \text{var}(return_3) \]
\[ + 2x_1x_2 \text{cov}(return_1, return_2) + 2x_1x_3 \text{cov}(return_1, return_3) \]
\[ + 2x_2x_3 \text{cov}(return_2, return_3) \]

(where \( \sum_{i=1}^{3} x_i = 1 \))
Efficient Portfolio Frontier

Efficient Portfolio Frontier With and Without Oil

- 28% Oil, 115% Stocks, -44% Bonds
- 21% Oil 79% Stock
- 15% Oil, 53% Stocks, 32% Bonds
- Tangency: Rf=5%, 12% Oil, 36% Stocks, 52% Bonds
- 50% Stocks, 50% Bonds
- 9% Oil, 27% Stocks, 64% Bonds
- 25 Stocks, 75% Bonds
- 100% Stocks
- 100% Bonds

Expected Annual Return

Standard Deviation of Annual Return

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Oil Reserves vs. Pension Fund Assets, 2006
Beta

- The CAPM implies that the expected return on the ith asset is determined from its beta.
- Beta ($\beta_i$) is the regression slope coefficient when the return on the ith asset is regressed on the return on the market.
- Fundamental equation of the CAPM:

$$r_i = r_f + \beta_i (r_m - r_f)$$
Survey of Individual Investors 1999

“Trying to time the market, to get out before it goes down and in before it goes up, is:

1. A smart thing to do; I can reasonably expect to be a success at it. 11%
2. Not a smart thing to do; I can’t reasonably expect to be a success at it. 83%
3. No opinion 5%
Survey of Individual Investors 1999

“Trying to pick individual stocks, for example, if and when Ford Motor stock will go up, or IBM stock will go up, is:

1. A smart thing to do; I can reasonably expect to be a success at it. 40%
2. Not a smart thing to do; I can’t reasonably expect to be a success at it. 51%
3. No opinion 8%
Survey of Individual Investors 1999

“Trying to pick mutual funds, trying to figure out which funds have experts who can themselves pick which stock will go up, is:

1. A smart thing to do; I can reasonably expect to be a success at it. 50%
2. Not a smart thing to do; I can’t reasonably expect to be a success at it. 27%
3. No opinion 23%