PS 13 Physics 201 April 21, 2010 R.Shankar Due April 28 in Natasa Mateljevic's Mailbox in Sloane Physics Lab

1. Consider a string fixed at $x = 0$ and $x = L$ whose displacement obeys
$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi(x,t)}{\partial t^2} \tag{1}$$
Assuming $\psi(x,t) = F(t)\psi(x)$, find the equation obeyed by F and by $\psi(x)$ and verify that a string that starts out at rest in a state $\psi(x,0) = A\sin\frac{n\pi x}{L}$ evolves into $\psi(x,t) = A\sin\frac{n\pi x}{L}\cos\frac{n\pi vt}{L}$.

2. Consider a free particle ($V(x) = 0$) moving in a ring of circumference L. Let $\psi_n(x)$ the normalized state of momentum $p = 2\pi n\hbar/L$. (i) Show that $\psi_n(x)$ is also state of definite energy E and find its energy by showing it satisfies $H\psi = E\psi$ where (in general)
$$H\psi(x) \equiv -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x).$$
(ii) Let $\psi(x,0) = 3\psi_2(x) + 4\psi_3(x)$. Normalize this state. Find $\psi(x,t)$ and $P(x,t)$.

3. (i) Recall the harmonic oscillator ($V(x) = \frac{1}{2}m\omega^2x^2$) has a ground state wave function
$$\psi_0(x) = \left[\frac{m\omega}{\pi\hbar}\right]^{\frac{1}{4}} e^{-m\omega x^2/2\hbar} \tag{2}$$
Sketch this function.

   (ii) Consider the function
$$\psi_1(x) = Axe^{-m\omega x^2/2\hbar} \tag{3}$$
Sketch it and find A to normalize this. (Use integrals from last PS if you want.)

   (iii) Show that it obeys $H\psi = E\psi$ and find the energy of this state.

   The allowed energies for the oscillator are
$$E_n = \left[n + \frac{1}{2}\right]\hbar\omega.$$
Thus the ground state corresponds to $n = 0$ and is thus labeled by $n = 0$ in Eq. 2, and likewise, the first excited state above is labeled by the subscript $n = 1$.

   (iv) Show that
$$\int_{-\infty}^{\infty} \psi_0(x)\psi_1(x)dx = 0 \tag{4}$$
(v) Suppose $\psi(x,0) = 3\psi_0(x) + 4\psi_1(x)$. Normalize this state. Find $\psi(x,t)$ and $P(x,t)$.

4. Complete the diagnostic survey found in the Tests and Quizzes section of the classes server. Click on the link that says: "End of Term Survey" to start. This 32 question survey is for us to get an objective idea of how effective our teaching methods were and what topics were covered. The survey will be available from Noon Wednesday April 21st to Noon Wednesday April 28.