

## PS 9 Physics 201 March 24, 2010 R.Shankar Due March 31.

1. You stand in front of a mirror whose top lines up with the top of your head (at which level we will assume your eyes are located.) How far down should the mirror extend so that you can see all the way down to your shoes? Express the answer in terms of your height  $h$ , using a figure to make your point.
2. A concave mirror has  $f = 30\text{cm}$ . At what distance  $u$  should I place an object so that the image distance  $v$  equals  $u$ ?
3. A concave mirror with  $f = 2\text{m}$  is lying on the ground with its vertex touching the ground. At  $t = 0$  a ball is dropped from a height of  $5\text{m}$  directly above the vertex. (i) Describe the image as a function of time till it hits the mirror for the first time assuming  $g = 10\text{m/s}^2$ . (ii) When will it pass the focal point for the first time? (iii) At what subsequent times will it hit the mirror again?
4. An object  $40\text{ cm}$  to the right of a lens forms an image  $10\text{ cm}$  from the lens on the other side. Find  $f$  and the magnification  $M$  (which is positive (negative) if the image is upright (inverted).)
5. An object is placed  $24\text{ cm}$  to the left of a diverging lens with  $f = -12\text{ cm}$ . A converging lens with  $f = 24\text{ cm}$  is placed  $d\text{ cm}$  to its right. Find  $d$  so that the final image is at infinity. (Hint: This is a two-stage problem).
6. Consider a convex lens with two different radii of curvature,  $R_1$  and  $R_2$ . Using the Principle of Least Time, adapt the proof given in class for  $R_1 = R_2$  to show that
 
$$\frac{1}{u} + \frac{1}{v} = \frac{(n-1)(R_1 + R_2)}{R_1 R_2}$$
7. A concave mirror with  $f = .6\text{m}$  has an image for which  $v = u/3$ . Find  $u$  and  $v$ .
8. Argue that a convex mirror always produces a virtual, upright and smaller image.
9. Consider a horizontal slab of glass of thickness  $d$  and refractive index  $n$ . Show that a ray striking from the top at an angle  $\theta_1$  from the normal, exits the bottom face at the same angle, but displaced. Show that the displacement  $x = d \frac{\sin(\theta_1 - \theta_2)}{\cos \theta_2}$  where of course  $n_1 \sin \theta_1 = n_2 \sin \theta_2$ .
10. A converging lens with  $f = .08\text{ m}$  is placed  $.12\text{ m}$  to the right of another identical lens. An object is placed  $.04\text{ m}$  to the left of the left lens. Where is the final image relative to the right lens?
11. Here you want to show that if we define the ellipse to be the set of points the sum of whose distances  $r$  and  $r'$  from the two focal points,  $((\pm c, 0)$  in Fig. 1), is a constant, then it obeys
 
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where  $2a$  and  $2b$  are the semi-major and semi-minor axes.

(i) First show that  $r + r' = 2a$  by choosing the point  $P$  to coincide with  $(a, 0)$  in the figure. (ii) Next show that  $a^2 = b^2 + c^2$ . (iii) Then square both sides of  $r + r' = 2a$ , rearrange and square again to get rid of all square roots. To avoid tedious algebra, call the recurrent combination  $x^2 + y^2 + c^2$  as  $w$  and work with  $w$  and use its explicit form only towards the end.

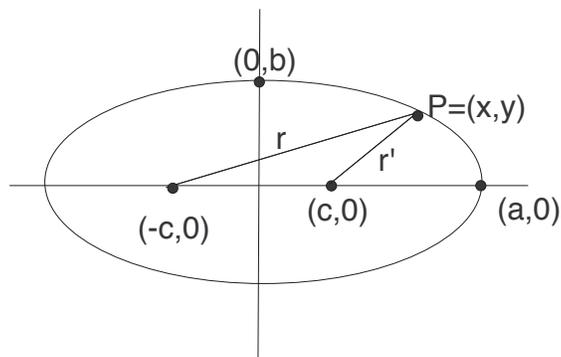


Figure 1: The ellipse is the set of points  $P = (x, y)$  the sum of whose distances  $r$  and  $r'$  from the focal points  $(\pm c, 0)$  is a constant.