Problem Set 3 Physics 201b January 27, 2010. Due Feb 3

1. Verify that the following force is conservative by applying the test:

$$\mathbf{F} = \mathbf{i}x^2y + \mathbf{j}\frac{x^3}{3}.$$

Find the corresponding potential $U(x, y)$ by inspection. What is

$$\int \mathbf{F} \cdot d\mathbf{r}$$

between the origin and the point $(x = 2, y = 3)$?

2. A 10 V battery stores $1.6 \cdot 10^3$ Joules of energy. How many Coulombs flow from one terminal to the other when it is fully discharged? (Assume the voltage remains the same during the discharge.) How many electrons is that (just get the powers of 10 right)?

3. A charge of $2\mu C$ is placed at the origin and a charge of $-3\mu C$ is placed at $(x = .2m, y = .5\ m)$. How much work is needed to move a $2\mu C$ charge from $(1m, 1m)$ to $(2m, 2m)$?

4. Find the electric field $\mathbf{E} = \mathbf{i}E_x + \mathbf{j}E_y$ in the x-y plane due to a dipole by differentiating the potential. Assume the dipole $\mathbf{p}$ points along the x-axis and is centered at the origin. Notice that E_x and E_y are just the negative of rates of change of V with distance in the x and y directions. Suppose we write $\mathbf{E} = \mathbf{e}_r E_r + \mathbf{e}_\theta E_\theta$ in polar coordinates in terms of unit vectors in the direction of increasing r and θ defined in Figure 4. Starting with $V(r, \theta)$ find $\mathbf{E}$ in polar coordinates.

5. Two charges q and $-2q$ are located at $(a, 0, 0)$ and $(0, 0, 0)$. Show that the surface $V = 0$ is a sphere and find it radius and center. Describe the image charge problem you can solve with this result.

6. Show that the potential due to a disc of radius R in the x-y plane (centered at the origin) with charge Q (uniformly distributed) at points on the z- axis is

$$V = \frac{Q}{2\pi\varepsilon_0 R^2}\left(\sqrt{z^2 + R^2} - |z|\right)$$

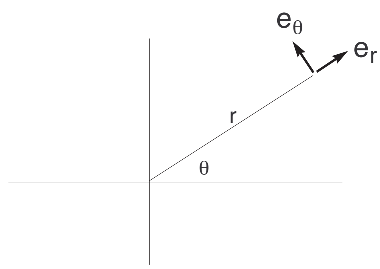

Figure 1: Polar coordinates for the x-y plane.

Show that this answer has the right limits for $z \to \pm\infty$ and $z \to \pm 0$. Some books incorrectly give the second term as z instead of $|z|$. Compute E_z on the z-axis and show that this leads to the wrong answer by considering E as $z \to \pm 0$. Another common mistake: you cannot conclude E_x and E_y vanish on the axis just because $-\frac{\partial V}{\partial x} = -\frac{\partial V}{\partial y} = 0$ for the V above. Why? But we do know $E_x = E_y = 0$ from direct computation of **E** using symmetry. What is the proper way to establish this using V? (No need to do the calculation, just tell me what you need to do.)

7. Given that $V(z)$ is 120V and 100V at distances $z = 1m$ and $z = 2m$ in the previous problem, find R and Q. (Note: V stands for volts as well as potential.)

8. A spherical charge distribution of radius R has a charge Q uniformly distributed over its volume. Find the potential $V(r)\ \forall\ r$.

9. Find the work it takes to assemble a sphere of radius R and charge Q by summing over the work to add a shell of thickness dr on top of a sphere of radius r using any result from the previous problem. Verify that this equals the volume integral of the field energy density $\frac{\varepsilon_0 E^2}{2}$.

10. Consider an infinitely long hollow conducting cylinder of radius a and charge λ per unit length surrounded by an outer hollow conducting cylinder of radius b with charge $-\lambda$ per unit length. Find $V(r)\ \forall r$, where r is the radial distance from the axis.

11. Two spheres of radius $r_1 = 10cm$ and $r_2 = 20cm$ carry charges 30 nC and $-20nC$ respectively. They are very far apart. What is the potential difference between them? If they are connected by a conducing wire, what will be the final potentials and charges on each?

12. Find the capacitance of two coaxial cylinders of length L and radii $a < b$ treating them as infinitely long for the purposes of computing **E** in the region between them. Compare to a parallel plate capacitor when $b - a = d <<< a$ or b.