Problem Set 9.

Econ 159a/MGT522a   Ben Polak

Three Problems (and one optional problem) due November 28, 2007

1. War of Attrition. In the *two-stage* war of attrition we studied in class, we focussed on the case where $v > c$. For the converse case $c > v$, find all the sub-game perfect equilibria, pure and mixed, including equilibria in which players mix in the first stage but do not mix in the second. [Hint: be careful when calculating the mixing probabilities to take account of what will happen in stage 2].

2. **"It's a Wonderful Life"**. Arnold and Boris have each deposited the same amount, $\$D$, into the Smalltown Saving and Loan (S&L). Since Arnold and Boris are the only two investors, the initial total in the S&L is $\$2D$. Money in the S&L grows over time: perhaps, the nice manager of the S&L invests it in local people's businesses and housing. By period 1, the total money in the S&L will increase to $2r$. If the S&L survives to period 2, the money will increase to $2R$. Assume $R > r > D$.

In period 1, each depositor can either withdraw or not-withdraw his money. (He cannot withdraw just part of it). These choices are made simultaneously. If either of them withdraws, the S&L goes bust. If just one person withdraws in period 1, the withdrawer gets $\$(2r - D)$. In this case, the non-withdrawer gets only his initial deposit $\$D$. If both withdraw in period 1, each gets $\$r$.

If neither depositor withdraws in period 1 then the S&L survives into period 2. Once again, each of them can either withdraw or not-withdraw their money. As before, these choices are made simultaneously. The S&L closes down at the end of period 2 regardless. If just one person withdraws in period 2, the withdrawer gets $\$(2R - D)$, and the non-withdrawer gets $\$D$. If neither withdraw in period 2 or if both withdraw in period 2, each gets $\$R$.

(a) Write out the extensive-form (game tree) of this game taking care to indicate what nodes lie in the same information sets.

(b) Consider two cases. Where $R + D > 2r$, and where $R + D < 2r$. In each case, find all the pure-strategy subgame perfect equilibria (SPE), either explaining why they are SPE, or (if you prefer) showing how you constructed them.

(c) Assume now that $R+D > 2r$, and suppose that the manager of Smalltown S&L guarantees that the smallest amount a depositor can get in the event that the S&L goes bust in period 1 is $\$r$. Perhaps he does this by offering up some cash of from his own pocket. Is there still an SPE in which the S&L goes bust in period 1? In what sense does the guarantee reduce the 'likelihood' of the S&L going bust in period 1?

3 Dating in the 'Burbs. Cheryl and Derrick (a.k.a. our hapless couple) are still trying to go out on their date. Cheryl can choose either to stay at home and read a book, or to go to the movies and perhaps meet up with Derrick. If she reads a book, her payoff is $\frac{3}{4}$ and Derrick's payoff is 0. If she chooses to go to the movies then Derrick observes that Cheryl is going to the movies but does not observe which movie Cheryl chooses. The two possible movies are accurate portraits of typical suburban life in the USA: "American Beauty" [excellent] and "Arlington Road" [okay]. If Cheryl goes to the movies, the payoffs are as follows:

		Derick	
		Beauty	Road
Cheryl	Beauty	$k, 1$	$0, 0$
	Road	$0, 0$	$1, 2$

where $k > 1$.

(a) Draw the extensive form (game tree) and normal form (matrix) of this game.

(b) For what values of k [recall, $k > 1$], will *all* the subgame-perfect equilibria involve Cheryl's going to the movies?

4. [Optional] Burning Money. (Ben-Porath & Dekel via Grant) Rich and Jo are expected dollar maximizers and are about to play a "Battle of the Sexes" game in which the payoffs in dollars are as follows:

		Jo	
		L	R
Rich	U	$18, 12$	$0, 8$
	D	$8, 0$	$12, 18$

Before playing the game, the only communication allowed is that Rich can, if he chooses, take a five dollar note from his pocket and burn it. Jo passively observes this choice before making choosing between L or R.

(a) Draw the extensive form for this game, including the burn or not burn stage. [Take care to identify information sets and to subtract $5 from Rich's payoff where relevant]. What are the subgames? What are the strategies for each player? Write down the normal form version of the game. [For the purpose of this question, if you prefer, you can ignore Rich's redundant strategies. For example, if a strategy of Rich's instructs him not to burn the money, you need not worry what it instructs him to do after burning the money. But each strategy for Jo must still specify an action at each of her information sets.]

(b) Find all the pure-strategy Nash equilibria of the game.

(c) Use iterative deletion of strongly and weakly dominated strategies on the game. What is the outcome? Did the money get burned? Was the ability to burn money useful? Briefly discuss whether deleting Burn-Down seems reasonable. Does this example shake your belief in iterative deletion of weakly dominated strategies?