Problem Set 7.

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Three questions due Nov 7, 2005 (plus one optional question).

1. "Not Survivor". [*This question was on the 2001 final exam.*] Six former 159a students Ann, Bob, Carl, Dora, Ed, and Fran are the contestants in a new "real TV" show. They are placed on an island. The rules of the game are as follows. Ann goes first. She is given a bag that everyone knows contains six gold coins. Ann makes a proposal of how to allocate the six coins among the six contestants including herself. The contestants (including Ann) then vote 'yes' or 'no' on the proposal. If the proposal gets *more* than half the votes then the coins are allocated according to the proposal and everyone leaves the island. If the proposal gets half or fewer than half the votes then Ann has to leave the island empty-handed and she is out of the game.

In this case, the bag of six gold coins passes to Bob. He gets to make a proposal of how to allocate the coins among the remaining contestants (i.e., including Bob but excluding Ann) and the remaining contestants (i.e., including Bob but excluding Ann) then vote. As before, if the proposal gets *more* than half the votes then the coins are allocated according to the proposal and everyone leaves the island. If the proposal gets half or fewer than half the votes then Bob has to leave the island empty-handed and is out of the game. In this case, the bag of six gold coins passes to Carl. And so on, with the same voting rules, with each failed proposal leading to expulsion of the proposer, and with the role of proposer being passed on alphabetically.

The following assumptions matter. The coins are indivisible, there is no other money on the island, and side contracts to make payments off the island are not allowed. *There are no abstentions; each surviving voter must vote yes or no: whenever a voter is indifferent, she or he votes no.* The players only care about the gold (and this is common knowledge). For example, leaving empty handed because your proposal fails is the same as leaving empty-handed because a successful proposal gives you no coins. Finally: it is common knowledge that all the contestants were well trained in Game Theory at Yale.

What proposal should Ann make and why?

2. A Hold-Up Problem. [*This question introduces you to an important kind of problem in sequential strategic settings.*] Suppose that Amtrack is choosing whether or not to build a new high-speed railroad on the east coast. Building the railroad will involve an initial up-front *sunk cost k*. To keep the accounting simple assume that the railway, if built, will run for exactly one year, and that it will generate (new) revenues of $130,000,000. Operating the railroad for that year would cost $10,000,000 in fuel, plus some labor costs. The labor costs depend on the wage. The railroad would need to employ 1000 workers all of whom would be unionized. The current

going wage for union rail labor on the east coast is $50,000. That is, without the new railroad, these workers would earn $50,000.

(a) Very briefly define what is meant by a sunk cost, and what is meant by the 'sunk-cost fallacy'. [Go look it up if you do not know.]

(b) Assuming that the labor can be hired at this going wage, for what values of k should Amtrack build the new railroad? (Assume that Amtrack aims to maximize profits without discounting).

(c) Suppose that, if the railroad is built, after it is built the rail union can make a 'take it or leave it' wage demand w to Amtrack to apply just for labor on the new line. The railroad's only choice is to accept to pay the wage demand, w, or close the new line down. What demand will the union make? Given this, if you were Amtrack, for what values of k would you build the new line? Why is your answer different from that in part (b)?

Now suppose that the wage demand made after the railroad is built is not a 'take it or leave it' demand but rather part of negotiation. Suppose that, fearing strikes in the transport sector, the government has instituted compulsory arbitration in wage disputes. The arbitrator always follows a two-step approach. First, she disqualifies any wage offers lower than the current going wage (that is, such that employees would rather walk away than accept the offer), and also any wage demand that would cause the employer to shut down (that is, such that the employee would rather walk away than accept the demand). Provided the offers and demands survive this test, she then "splits the difference".

(d) What wage demands and wage offers will be presented to the arbitrator after the railroad is built. Given this, if you were Amtrack, for what values of k would you build the new line? Why is your answer different from that in parts (b) and (c)?

(e) Issues like this are sometimes called 'hold-up' problems. One way to avoid the problem here (under-investment) is to give all the ex post bargaining power to the would-be ex ante investor. (Here this is the employer but in other settings it might be an employee who must decide how much to invest in skills that are specific to a given firm.) Briefly list some other ways we see people try to get around hold-up problems.

3. A Patent Race (Dutta) [*This question looks at a patent race under very different assumptions to the 'duel' we played in class.*] Two firms, A and B, are each considering trying to develop a new widget. Whichever firm is first to develop the new widget wins a patent worth $20 million plus a penny. Developing a new widget involves several 'steps'. The firms alternate moves, with A moving first, until one of them wins the patent. All moves are observed. In each turn, a firm can choose whether to take 0, 1, or 2 development 'steps'. Taking 0 steps in a turn costs that firm $0. Taking 1 step in a turn costs $4 million. And taking 2 steps in a turn costs $11 million. For simplicity, assume a zero discount rate. Initially, each firm is 4 steps away from completing development.

(a) Describe and explain carefully what will happen in this patent race and why. [Hint: it may help to read Dutta ch 12 (but notice I changed the numbers).]

(b) Very briefly explain what is the economic rationale for granting 'intellectual property

rights' such as patents. What are some disadvantages for society of granting such rights?

4. [Optional]. The Rectangular Array of Stones. Recall the game we played in class in which, initially, there is an $N \times M$ rectangle of stones. There are two players who take turns. The player whose turn it is to move must select a stone and remove that stone and that lie above or/and to the right of it. The loser is the person who takes the last stone. For all N and M, show what is the solution of the game: that is, can player 1 force a win or can player 2 force a loss. Try for as concise an argument as possible.