Final Exam

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- This is a **closed-book** exam.
- There are **6** pages including this one.
- The exam lasts for **150** minutes (plus **30** minutes reading time).
- There are **150** total points available.
- There are **five** questions, worth **20, 15, 40, 30 and 45 points** respectively.
- Please notice that there are **FORTY-FIVE** points available in the last question.
- Please remember to attempt the easier parts of all the questions. Do not get bogged down on the hard parts: **move on**!
- Please put **each** question into a **different blue book.**
- Show your work.
- Good luck!

USE BLUE BOOK 1

Question 1. [**20 total points**] State whether each of the following claims is true or false (or can not be determined). For each, explain your answer in (at most) **one** short paragraph**.** Each part is worth **5** points**, of which 4 points are for the explanation.** Explaining an example or a counter-example is sufficient. Absent this, a nice concise intuition is sufficient: you do not need to provide a formal proof. Points will be deducted for incorrect explanations.

(a) [**5 points**] *"William the Conqueror burned his boats because his soldiers were afraid of the dark."*

(b) [**5 points**] *"Consider the strategy profile* (s_A, s_B)*. If player A has no strictly profitable pure-strategy deviation then she has no strictly profitable mixed-strategy deviation.*

(c) [**5 points**] *"In duel (the game with the sponges) if your probability of hitting if you shoot now plus the probability of your opponent hitting if she were to shoot next turn is greater than one, then it is a dominant strategy for you to shoot now."*

(d) [**5 points**] *"Lowering the tuition to go to elite schools like Harvard and Yale makes it harder for bright students to distinguish themselves from less bright students."*

USE BLUE BOOK 1

USE BLUE BOOK 2

Question 2. [**15 total points**]

Two players, A and B play the following game. First A must choose IN or OUT. If A chooses OUT the game ends, and the payoffs are A gets 2, and B gets 0. If A chooses IN then B observes this and must then choose in or out. If B chooses out the game ends, and the payoffs are B gets 2, and A gets 0. If A chooses IN and B chooses in then they play the following simultaneous move game:

		B	
		left	right
A	up	$3, 1$	$0, -2$
	down	$-1, 2$	$1, 3$

(a) [**5 points**] Draw the tree that represents this game?

(b) [**10 points**] Find all the *pure-strategy* SPE of the game.

USE BLUE BOOK 2

USE BLUE BOOK 3

Question 3. **[40 total points] Poverty Traps.**

Alex is deciding whether or not to make a loan to Brian who is very poor and who has a bad credit history. Simultaneous to Alex making this decision, Brian must decide whether or not to buy gifts for his grandkids. If he buys gifts, he will be unable to repay the loan. If he does not buy gifts, he will repay the loan. If Alex refuses to give Brian a loan, then Brian will have to go to a loan shark.

The payoffs in this game are as follows: if Alex refuses to make a loan to Brian and Brian buys gifts then both Alex and Brian get 0. If Alex refuses to make a loan to Brian and Brian does not buy gifts then Alex gets 0 and Brian gets -1. If Alex makes a loan to Brian and Brian buys gifts then Alex gets -2 and Brian gets 7. If Alex makes a loan to Brian and does not buy gifts, then Alex gets a payoff of 3 and Brian gets a payoff of 5.

(a) [**5 points**] Suppose this game is played just once. Find the equilibria of the game.

Now suppose that the game is repeated. Suppose that (for all players) a dollar tomorrow is worth 2/3 of a dollar today. In addition, suppose that, after each period (and regardless of what happened in the period), Brian has a 1/2 chance of escaping poverty. Assume that, if Brian escapes poverty then he will not need a loan from either Alex or a loan shark: if effect, Brian will exit the game. Assume that, if Brian escapes poverty, he will never return. Thus, after each period, there is only 1/2 chance of the game continuing. Given this, the effective discount factor for the game between Alex and Brian is $(1/2) \times (2/3) = (1/3)$.

Consider the following strategy profile. In period one, Alex makes Brian a loan. Thereafter, Alex continues to make Brian loans (if he is still poor) as long as Brian and has always got a loan and repaid it in the past. But if Brian ever does not repay (or does not get a loan) then Alex never makes a loan to Brian again. In period one, Brian does not buy gifts (and hence repays the loan if he gets one). Thereafter (as long as he is still poor), Brian does not buy gifts (and hence repays the loan if he gets one) as long as he has always got a loan and repaid it in the past. But if Brian ever does not repay (or does not get a loan) then he will return to buying gifts and hence never repay a loan again.

(b) [**12 points**] Is this strategy profile an SPE of the repeated game?

(c) [**8 points**] Suppose that the government introduces regulation of loan sharks. As a consequence, Brian's payoff in each period in which he still needs a loan but does **not** get it from Alex is 1 if he does not buys gifts and 2 if he buys gifts. Explain whether or not this policy is likely to be good for Brian.

(d) [**8 points**] Suppose that the government abandons its loan-shark policy and replaces it with a job scheme that increases the probability after each period of Brian escaping poverty to 2/3 (i.e., 1/3 chance of returning to the loan game). Explain the likely consequences of this policy for the business relationship between Alex and Brian.

(e) [**7 points**] [Harder] For the policy in part (d) what extra information would you need to know whether this policy is good or bad for Brian (ignoring the welfare of Alex or Brian's grandkids). Explain as carefully as you can. [**Do not spend all your time on this: you can come back later.**]

USE BLUE BOOK 3

USE BLUE BOOK 4

Question 4. [**30 total points**] "**Exclusive**".

The Europa Club has a formal procedure (which we can think of as a game) to select its members. At each stage of the game, the 'newest member' to have been admitted into the club can either declare the membership-game over or nominate a new candidate to become a member. If a candidate is nominated, the existing members of the club vote whether to admit or reject. If the candidate is rejected, then the membership game is over. If the candidate is admitted, then the game continues with the now-admitted candidate becoming the 'newest member' choosing whether to nominate someone or end the game. The final membership of the club are the members when the game is over.

Whenever votes occur, the voting rules are as follows. The existing members vote sequentially, starting with the newest member (assume the nomination is his vote) and ending with the first member. The candidate does **not** get a vote. All votes are observed by everyone. If the candidate gets a half or more of the votes, she is admitted. That is, if there is a tie, then the candidate is admitted. There are no abstentions. Once you become a member, you are a member for ever: you cannot be voted off and you cannot leave.

Suppose initially that A is the only member of the club (and hence also its newest member). There are only three possible other members: B, C and D. Thus, in the first stage of the game, A can either nominate one of these as a candidate (and then 'vote' them in), or end the game and remain alone.

The following table gives the preferences of each possible member over possible final memberships of the club.

A	B	C	D
ac	$abcd$	acd	abd
ab	ab	ac	ad
ad	abd	$abcd$	acd
a	abc	abc	$abcd$
$abcd$	a	a	a
abc	ac	ab	ab
acd	ad	ad	ac
abd	acd	abd	abc

Thus, for example, B's most preferred final membership would have everyone in the club. Her second preference would be just A and herself. Her third preference would be A, D and herself. And her fourth preference would be AC and herself. All other memberships rank lower in her preferences.

(a) [**5 points**] Suppose three candidates have been admitted, and a fourth has been nominated. How will player A vote? Explain why this means that any nominated member will be admitted.

(b) [**25 points**] Assuming that all members have taken game theory, explain carefully how you would expect the game to proceed. [**Most of the points are for the explanation.**]

USE BLUE BOOK 4

Question 5. [**45 total points**] **"Paid in the USA"** [Notice that you do not need to know any auction theory to answer this question.]

Two interest groups, A and B, are lobbying congress about an upcoming bill. Everyone knows that it is worth \$3M to A to get the bill passed into law, and it is worth \$2M to B to get the bill to fail. Congress decides to sell off its vote using a sealed-bid second-price auction. That is, A and B simultaneously write down a 'bid'. The bids, b_A and b_B are then 'opened'. If $b_A \geq b_B$ then congress passes the bill and A must pay b_B to the "congressmen's fund". If $b_B > b_A$ then congress rejects the bill and B must pay b_A to the "congressmen's fund". Notice: if the bids are tied then A 'wins'; **the winner pays the loser's bid;** and the loser pays nothing.

(a) [**10 points**] Recall from class that bidding your value is a weakly dominant strategy in a second-price auction. Argue carefully but concisely that, for A, bidding \$3M weakly dominates bidding \$2.8M.

(b) [**5 points**] Assuming that no-one chooses a weakly dominated strategy, what are the equilibrium bids, payments and payoffs in the auction?

Now suppose that, if and only if congress passes the bill, it goes to the president. If the president signs the bill, it passes into law. If he vetoes it, it fails. The president decides that, if the bill gets to him, he will also hold a sealed-bid, second-price auction under exactly the same rules except that payments are made to the "president's fund". That is, there are potentially two auctions, held sequentially. The first auction decides whether or not congress passes the bill with payments made to congress accordingly. Then, afterwards (if congress passes the bill), a second, **new** auction decides whether or not the president signs the bill with payments to the president accordingly.

(c) [**5 points**] Suppose that A wins the first auction, and the bill passes congress (with A paying, say, $b_B = \$0.9M$ to the congressmen's fund). Assuming that no-one chooses a weakly dominated strategy in the subsequent presidential auction, what are the equilibrium bids, payments and *continuation* payoffs in that auction?

(d) [**7 points**] Assuming that no-one chooses a weakly dominated strategy in any subgame, explain the SPE outcome of the whole game including the first-stage bids and payments.

Now suppose that congress (realizing that A is deterred from bidding much in the first auction) makes the following offer only to A. If A 'wins' the congressional auction (so that congress passes the bill and A pays B's bid b_B to congress) but the bill is then vetoed by the president (that is, A loses the second auction), then congress will refund to A its payment b_B minus a small processing fee.

(e) [**10 points**] Suppose that A wins the first auction, and the bill passes congress with A paying $b_B = \$0.9M$ to the congressmen's fund. Assuming that no-one chooses a weakly dominated strategy in the subsequent presidential auction, what are the equilibrium bids and payments in that auction? How would your answer change if A had paid $b_B = \$1.1M$ in the first stage?

(f) [**8 points**] [Harder] Assuming that no-one chooses a weakly dominated strategy in any subgame, explain the SPE outcome of the whole game including the first-stage bids and payments.