Physics 200a Finals 18 December 2006
180 minutes
Formulas and Figures at the end.
Do problems in 6 books as indicated.

(I) **Book 1** A solid cylinder of mass m and radius r rolls (without slipping) down a slope from a height h and goes around a loop-the-loop of radius R as in figure 1. What is h, the minimum height from which it must be released to make the trip around the loop? You may neglect r in comparison to R and h. (**20**)

(II) **Book 1** A record is spinning on a turntable at 8 rev/s. A non-rotating rod of equal mass is dropped onto it from negligible height. The rod's length equals the diameter of the record. They begin to turn as one, with their centers coincident. What is the final rate of revolution in rev/s ? What is r, the ratio of the final to initial kinetic energy of the system? If you use any conservation law(s) justify it (them). (**20**)

(III) **Book 2** A solid cylinder rolls without slipping on an incline of angle θ. Show that its linear acceleration downhill is

$$a = \frac{2}{3}g\sin\theta.$$

(Hint: Take torque about a judicious point.) (**20**)

(IV) **Book 2** An amusement park ride is shown in figure 2. The terrified customer stands with the back against the wall of the rotating cylindrical drum. Once it picks up enough angular speed, the floor drops out but customer does not fall down. (i) Draw the forces on customer (ii) Write down $F = ma$ in two independent directions. (iii) Show that the minimum frequency of the drum is

$$f_{min} = \frac{1}{2\pi}\sqrt{\frac{g}{\mu_s R}}$$

where R is its radius μ_s is the coefficient of static friction between the rider and the drum. (**15**)

(V) **Book 3** A driven oscillator is vibrating according to

$$m\frac{d^2x}{dt^2} + m\gamma\frac{dx}{dt} + m\omega_0^2 x = F_0\cos\omega t.$$

Using complex numbers derive $x(t)$, the *steady state solution* in terms of F_0, m, and ω_0 for the case $\omega = 2\omega_0$ and $\gamma^2 = \frac{7}{4}\omega_0^2$. The only information I am giving you for this problem is

$$e^{i\theta} = \cos\theta + i\sin\theta. \quad (\mathbf{20})$$

(VI) **Book 3** A vibrating string (clamped at both ends) has a fundamental frequency 440 Hz is 50 cm long and under a tension of $F = 700$ Newtons. What is its mass? ($\mathbf{10}$)

(VII) **Book 4** A 400 kg raft floats on a lake. When a 45 kg man steps on it, it sinks 4.5 cm deeper into the water. When he steps off, it vibrates up and down. Calculate the frequency, amplitude, energy and maximum acceleration of oscillation. (Ignore any frictional or dissipative effects. Start by finding k.) ($\mathbf{15}$)

(VIII) **Book 4** A clock moving at velocity $u = 3c/5$ passes me, sitting at my origin, at $t = t' = 0$ according to it and my clock. What is its location in my frame when it ticks 1 second in its frame? If it emits alight pulse at that time, at what time t^* according to me will that pulse reach my origin? Use (x, t) for me and (x', t') for clock frame. ($\mathbf{15}$)

(IX) **Book 5** (In this problem use $c = 1$). A right moving photon with $K = (\omega, k)$ hits an electron at rest, i.e., electron has $P = (m, 0)$. The photon bounces back with energy ω' while the electron recoils to the right. Show that

$$\frac{1}{\omega'} = \frac{1}{\omega} + \frac{2}{m}$$

Hint: Use four vectors and take the square of the least known four-vector. ($\mathbf{20}$)

(X) **Book 6** A rod of rest length L_0 is moving at speed v relative to me. Show that its length according to a person moving to the right at velocity u relative to me is (with c=1)

$$L = L_0 \frac{\sqrt{(1 - u^2)(1 - v^2)}}{(1 - uv)}.$$

Test your result at $u = v$. ($\mathbf{5}$)

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(XI) **Book 6** One mole of a monatomic gas goes in a cycle ABCA as in figure 3. Let (P_0, V_0, T_0) be the parameters at A and $2T_0$ the temperature at B. Find (i) the values of T_C and P_B (ii) the work down in each part: AB, BC and CA in units of RT_0. (iii) Repeat for heat input in each part. (iv) Find the entropy change in each part. You may use the fact that entropy is a state variable to save some work. (**20**)

Data Sheet

$$c = 3 \cdot 10^8 \ m/s \quad g = 9.8 m/s^2$$

$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}} \quad t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}}$$

$$A \cdot B = A_0 B_0 - \overline{\mathbf{A} \cdot \mathbf{B}} \quad \text{dot product of four vectors A and B}$$

$$P = m_0 \left(\frac{1}{\sqrt{1 - v^2}}, \frac{\mathbf{v}}{\sqrt{1 - v^2}} \right), \quad P \cdot P = E^2 - p^2 = m_0^2, \quad \text{c=1 here}$$

$$K = (\omega, k) \text{ for photons with } K \cdot K = w^2 - k^2 = 0$$

$$w = \frac{u - v}{1 - uv/c^2}$$

$$L = L_0 \sqrt{1 - u^2/c^2}$$

$$\tau = \frac{\tau_0}{\sqrt{1 - u^2/c^2}}$$

$$\text{Spacetime interval} = s^2 = (ct)^2 - x^2$$

$$A_1 v_1 = A_2 v_2 \quad P + \frac{1}{2} \rho v^2 + \rho g h = constant$$

$$\rho_{water} = 10^3 kg/m^3$$

$$\tau = I\alpha \quad \tau = Fr \sin \theta$$

$$I = \sum_j m_j r_j^2 \quad I_{discCM} = I_{cylinder} = \frac{1}{2} MR^2$$

$$I_{rod}^{CM} = \frac{1}{12} ML^2$$

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$$I = I_{CM} + Md^2$$

$$v = \sqrt{T/\mu} \quad v = \lambda f$$

$$c_{water} = 1\ cal/(g^oC) \qquad 1\ cal = 4.2J$$

$$\Delta Q = mC\Delta T \text{ for heating} \quad \Delta Q = mL \text{ for phase change, } L \text{ is latent heat}$$

$$PV = nRT \quad R = 2\ cal/mole \cdot {}^oC \qquad 1\ cal = 4.2J$$

$$U = \frac{3}{2}NkT = \frac{3}{2}nRT = \frac{3}{2}PV$$

$$W_{Adiabatic} = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$\Delta U = \Delta Q - PdV \qquad \text{Law I}$$

$$C_V = \left.\frac{dQ}{dT}\right|_{per\ mole\ V\ fixed} \qquad C_P = \left.\frac{dQ}{dT}\right|_{per\ mole\ P\ fixed}$$

$$C_P = C_V + R$$

$$W = \int_1^2 P(V)dV$$

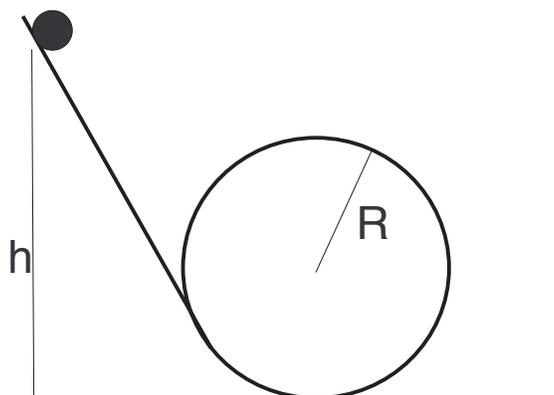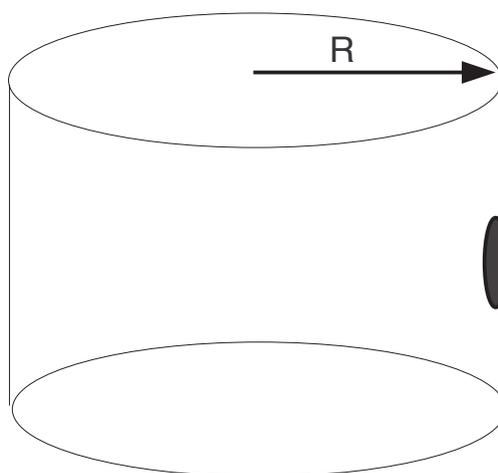$$\Delta S = \left.\frac{\Delta Q}{T}\right|_{reversible}$$

FIG. 1: A cylinder (solid circle) of radius r rolls down the loop-the-loop of radius $R \gg r$.

The petrified rider is the dark oval.

FIG. 2: A amusement park ride: as the drum picks up speed the floor drops off and petrified rider is held up in place by friction.
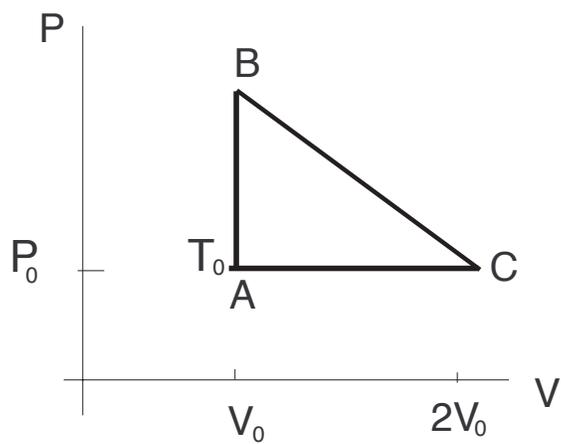
FIG. 3: A mole of monoatomic goes in a cycle ABCA. Let T_0 be the temperature at A.