Lecture 20   14 Nov 07

Last time apply SPE
- solve NE in each subgame
- roll back payoffs

Lesson Strategic effects matter!
- investment game
- tax design
- tolls

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2 players each period each chooses F or Q
game ends as soon as someone Q's

good news if the other player quits first, you win a prize $V = \$1$

bad news: each period in which both F, each player pay cost $-C = .75¢$

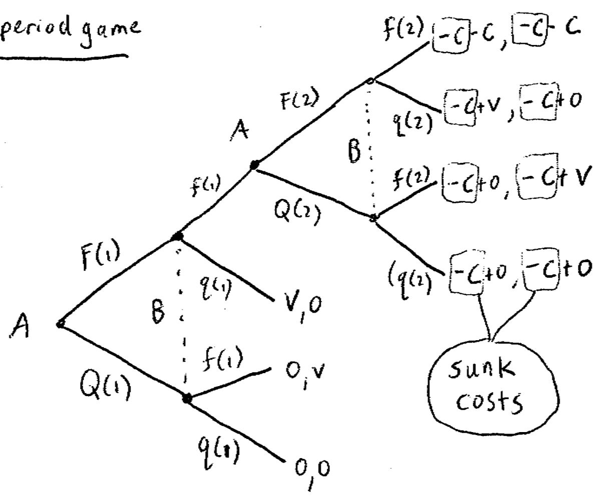
if both quit at once $\longrightarrow$ 0

examples
- WWI
- BSB v. Sky

$\boxed{\text{WARS OF ATTRITION}}$

- bribe contests

Two period game



Two cases: $\boxed{V > C}$ ← here in class
$V < C$ ← on homework

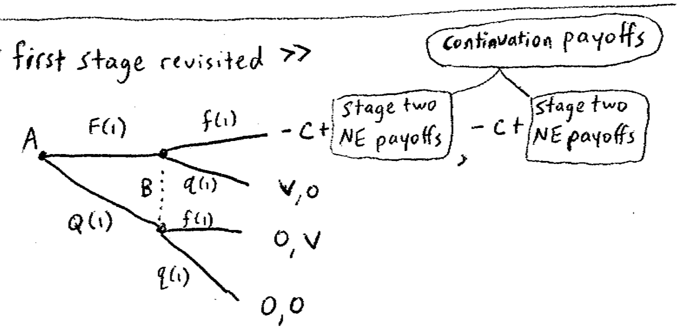
Second subgame

$\boxed{-C}$ + (SUNK COST)

A \ B	f(2)	q(2)
F(2)	−C, −C	V, 0
Q(2)	0, V	0, 0

Two pure-strategy NE in this subgame:
(F(2), q(2)) , (Q(2), f(2))
payoff (V, 0)   (0 , V)

« first stage revisited »



for (F(2), q(2)) in stage 2

A \ B	f(1)	q(1)
F(1)	−C+V, −C+0	V, 0
Q(1)	0, V	0, 0

NE (F(1), q(1))

for (Q(2), f(2)) in stage 2

A \ B	f(1)	q(1)
F(1)	−C+0, −C+V	V, 0
Q(1)	0, V	0, 0

NE (Q(1), f(1))

"If we know I'm going to win tomorrow, then I win today."

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Pure strategy SPE (with $v > c$)

$$[\ (F(1), F(2)),\ (q(1), q(2))\ ]$$

$$[\ (Q(1), Q(2)),\ (f(1), f(2))\ ]$$

"quitter v. fighter"

<< Now look for *mixed* strategy eq >>

Second subgame

$\boxed{-c}$ + A

SUNK COST

A \ B	f(2)	q(2)
F(2)	$-c, -c$	$\underline{v}, \underline{0}$
Q(2)	$\underline{0}, \underline{v}$	$0, 0$
	p	$(1-p)$

If A Fights ⟶ $-cp + v(1-p)$ $=$
If A Quits ⟶ $0p + 0(1-p)$

$v(1-p) = pc$

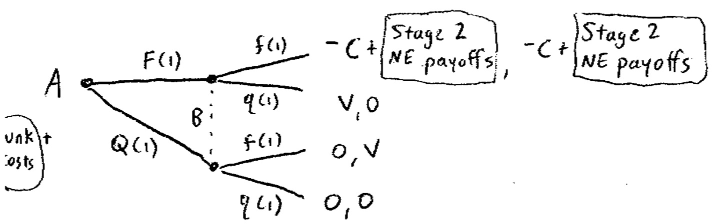
$p = \frac{v}{v+c}$

$1-p = \frac{c}{v+c}$

mixed NE has both fight with prob $= \frac{v}{v+c}$

payoffs in this mixed NE $= (0,0)$

<< back to first stage >>



A \ B	f(1)	q(1)
F(1)	$-c+0, -c+0$	$\underline{v}, \underline{0}$
Q(1)	$\underline{0}, \underline{v}$	$0, 0$
	p	$1-p$

For the mixed NE in period 2

<< same payoff matrix, so ... >>

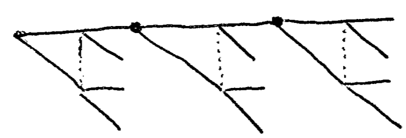
Mixed NE in this matrix is:
both F with prob $p^* = \frac{v}{v+c}$

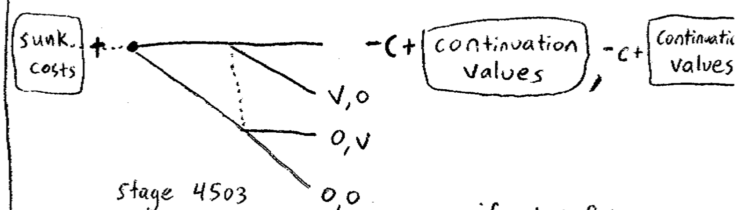
Mixed SPE $\boxed{[\ (p^*, p^*), (p^*, p^*)\ ]}$

E payoff is 0

<< Not pride, craziness >>
- Prob of Fights occuring $\uparrow$ in v $\downarrow$ in c

Infinite period game

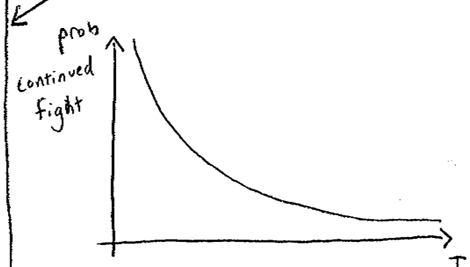




if mix in future, Cont. value (0,0)

<< Now this analysis is already solved! >>

Same conclusion, too: both mix with prob F $= p^* = \frac{v}{v+c}$



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