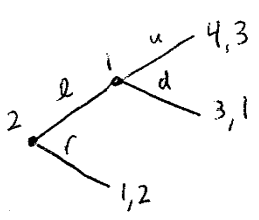
Lecture 19 12 Nov 07

last time
- info sets : imperfect information
- strategies : instructions for each info set
- subgames : game within games
- subgame perfection : NE in every subgame

Today Examples



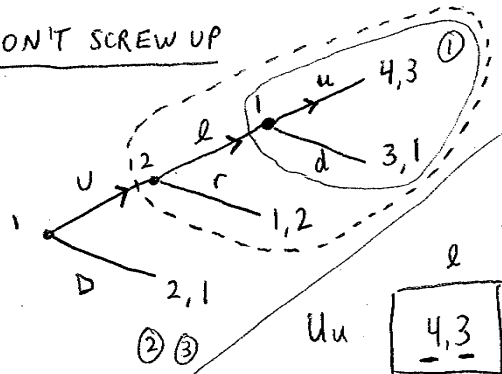
2	l	r
1 u	4,3	1,2
1 d	3,1	1,2

← pure NE
[u, l] [d, r]

①	②	③
[u, l]	[u, r]	[d, r]
✓	✗	✓

⇒ ② is eliminated since it induces non-NE play in this subgame

DON'T SCREW UP



2	l	r
Uu	4,3	1,2
Ud	3,1	1,2
Du	2,1	2,1
Dd	2,1	2,1

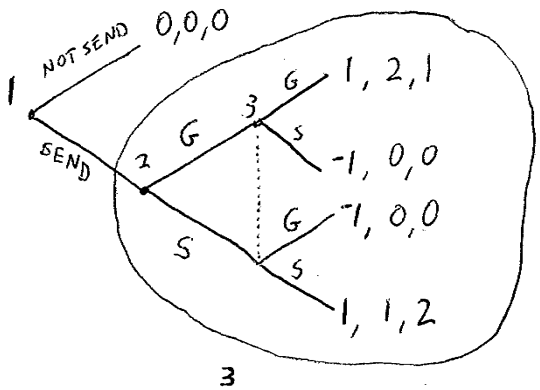
1

①	②	③
[Uu, l]	[Du, r]	[Dd, r]
BI	~~BI~~	~~BI~~

∴ The only SPE is ① [Uu, l]

NB This is the BI prediction.

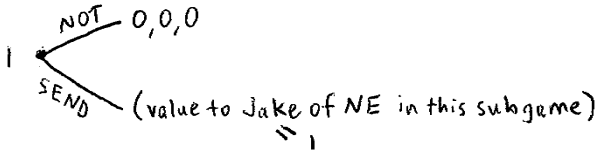
Matchmaker Game



3

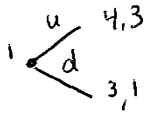
2 \ 3	G	S	
G	1, 2, 1	-1, 0, 0	2/3
S	-1, 0, 0	1, 1, 2	1/3
	1/3	2/3	

pure NE
(G,G) (S,S)
both yield a value of 1 for player 1



<<which subgames are SP NE? >>

To find SPE



1	
u	4,3
d	3,1

NE = [u]

①	②	③
u	u	d
✓	✓	✗

③ is eliminated because it induces play in this subgame that is not NE

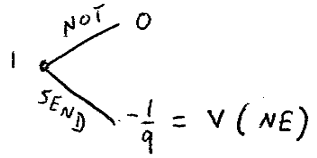
SPE = (Send, S, s)
(Send, G, G)

In the subgame, there is a mixed NE

$$\left[\left(\tfrac{2}{3}, \tfrac{1}{3}\right), \left(\tfrac{1}{3}, \tfrac{2}{3}\right)\right]$$

If 1 Jake sends 2 Dave and 3 Nina
then they meet with $\frac{2}{9} \cdot \frac{2}{9} = \frac{4}{9}$ probability
and hence fail to meet with prob $= \frac{5}{9}$

⇒ Value to Jake of this NE is $\frac{4}{9}[1] + \frac{5}{9}[-1] = -\frac{1}{9}$



SPE = (NOT, mix, mix)

Strategic Investment

2 firms Cournot

$P = 2 - \frac{1}{3}[q_A + q_B]$ millions of tons

MC C = $1 a ton

$q^* = \frac{a-c}{3b} = \frac{2-1}{3 \cdot \frac{1}{3}} = 1^M$ each firm (1 million tons of fertilizer)

$P^* = 2 - \frac{1}{3}[1+1] = \$1\frac{1}{3}$ per ton

profit: $[\$1\frac{1}{3} - \$1]\,1^M = \$\frac{1}{3}M$ per firm

new machine

- only works for A
- costs $.7M per year
- it lowers A's costs to 50¢ a ton

To Rent or not Rent?

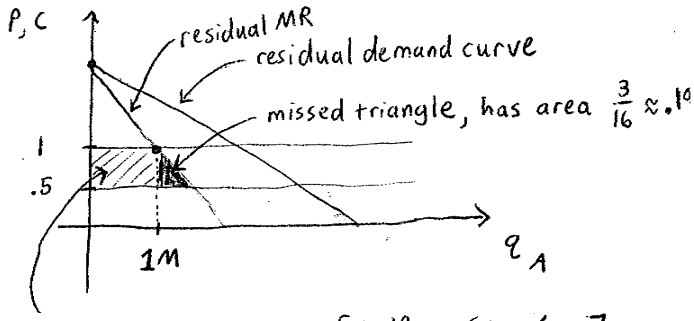
(1) Accountants Answer

produce 1M tons per year, save 50¢ per ton } save $.5M a year in variable cost

Cost of machin : fixed cost of $.7M

.5 < .7 ⇒ So NOT RENT

(2) Econ 115 Answer

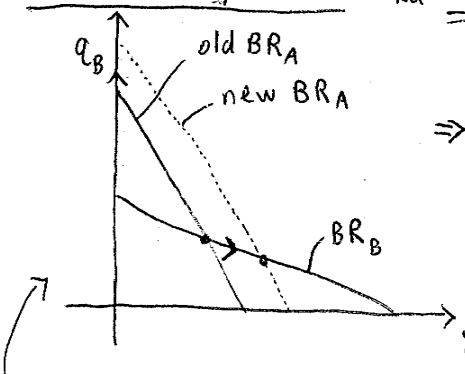


accounting answer .5 × 1

.5 + .19 = .69 < .7

So NOT RENT

(3) Game Theory Answer aka the right answer



$q_A \uparrow$ (econ answer)
⇒ $q_B \downarrow$ strat sub
This is good for firm A.

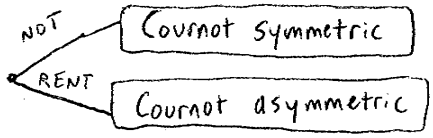
We could calculate the new NE subgame is BR-diagram above

<<do this at home>>

we get extra $.31^M + $.69^M = $1^M > $.7^M

RENT

Lessons: 1) solve out lower subgames, roll back



2) Strategic effect - don't forget them!
tax code, curriculum design