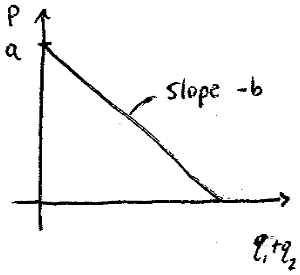
Lecture 14 24 Oct 07

Midterm out of 75

Mean 62

Quantity Competition Revisited - Stackelberg

Cournot $p = a - b(q_1+q_2)$

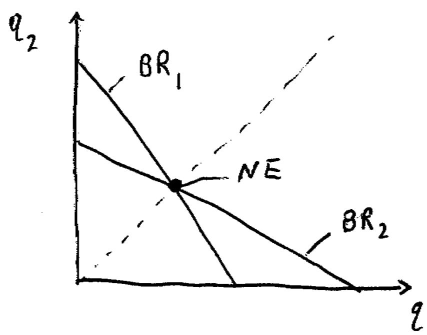


$u_1(q_1,q_2) = pq_1 - cq_1$

$BR_1(q_2) = \frac{a-c}{2b} - \frac{q_2}{2}$

$BR_2(q_1) = \frac{a-c}{2b} - \frac{q_1}{2}$

NE $\boxed{q^* = \frac{(a-c)}{3b}}$

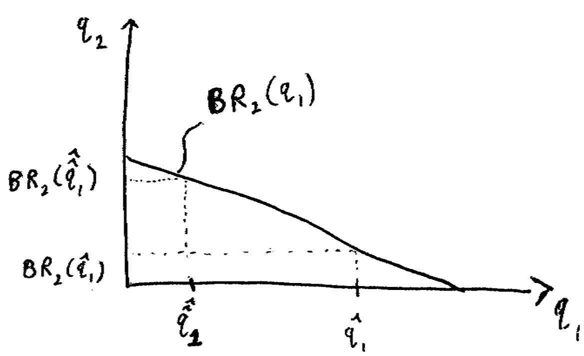


<< Stackelberg adds order to moves >>

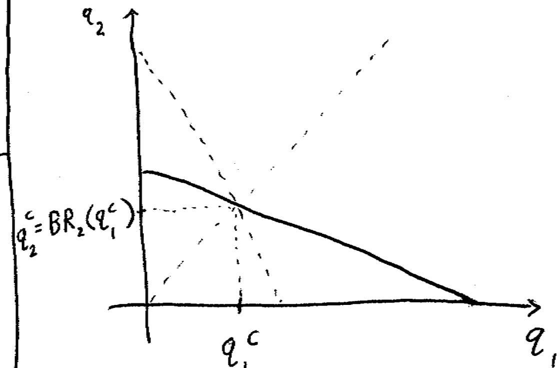
<< Is it an advantage to going first? >>

<< Use Backward Induction >>

Firm 2 sees q_1 and must choose q_2



By definition, $BR_2(q_1)$ tells us the profit-maximiz output of firm 2 taking q_1 as given



strategic substitutes : $q_1 \uparrow$ $BR_2(q_1)\downarrow$

$q_1\uparrow$ this suggests firm 1 should set $q_1 > q_1^c$

$q_2\downarrow$ to induce $q_2 < q_2^c$

firm 1's profits $\uparrow$

$(q_1+q_2)\uparrow$ $p\downarrow$ so firm 2 profit $\downarrow$

CS $\uparrow$

$p = a - b(q_1+q_2)$

$\text{Profit}_i = pq_i - cq_i$

BI solve for firm 2 first, taking q_1 as given

$\max_{q_2} [a - bq_1 - bq_2]q_2 - cq_2$

<< differentiate with respect to q_2, set to 0 >>

Solved $\rightarrow$ $\boxed{q_2 = \frac{a-c}{2b} - \frac{q_1}{2}}$

Now Solve for firm 1

$\max_{q_1} [a - bq_1 - bq_2]q_1 - cq_1$

$\max_{q_1} \left[a - bq_1 - b\left(\frac{a-c}{2b} - \frac{q_1}{2}\right) - c\right]q_1$

$\max_{q_1} \left[\frac{a-c}{2} - \frac{bq_1}{2}\right]q_1 = \frac{(a-c)}{2}q_1 - \frac{bq_1^2}{2}$

diff w.r.t. q_1

$\rightarrow \frac{a-c}{2} - bq_1 \stackrel{\text{set}}{=} 0$ First order condition

<< Second order condition ok : $\frac{\partial^2}{\partial q_1^2} = -b < 0$ >>

Open Yale courses

$$\boxed{q_1 = \frac{(a-c)}{2b}}$$

$$q_2 = \frac{a-c}{2b} - \frac{1}{2}\frac{(a-c)}{2b}$$

$$\boxed{q_2 = \frac{a-c}{4b}}$$

Check:

$q_{1\,new} > q_{1\,old} \; (= q^c)$ ✓

$q_{2\,new} < q_{2\,old} \; (= q^c)$ ✓

$\boxed{\underset{new}{q_1+q_2} = \frac{3(a-c)}{4b}} > \frac{2(a-c)}{3b} = \underset{old}{q_1^c+q_2^c}$ ✓

1) **Commitment** sunk costs can help

2) spy or having more information can hurt you
key: the other players knew you had more information << for it to hurt you >>
that information

reason: it can lead other players to take actions that hurt you

<< more info can hurt - spy
more options can hurt - Saxon army, collateral >>

3) FIRST-MOVER ADVANTAGE

<< • Yes sometimes. Stackelberg
- But not always. : Rock, Paper, Scissors
  Learning from Mistakes
  Second-mover advantage.
  Information here is helpful
- Sometimes neither first nor second mover adv.:
  "I split, you choose."

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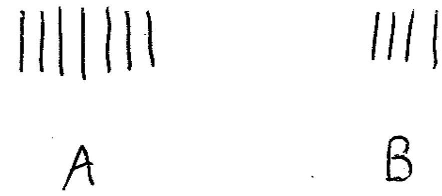

piles equal => second mover advantage
piles unequal => first mover advantage

1) you know who'll win from the initial setup
2) solve by backwards induction
3) can be first or second mover advantage within same game depending on setup