Lecture 12 15 Oct 07

Defn In a 2-player symmetric game, a strategy $\hat{s}$ in ES (in pure strategies) if

(a) $(\hat{s},\hat{s})$ is a symmetric NE, AND

(b) if $(\hat{s},\hat{s})$ is not strict NE,

[i.e. if there is an $s' \neq \hat{s}$ with $u(\hat{s},\hat{s}) = u(s',\hat{s})$]

then $u(\hat{s},s') > u(s',s')$

	a	b
a	1,1	1,1
b	1,1	0,0

<< what is Nash? >> (a,a) is sym. Nash

Is (a,a) strict NE? No: $u(a,a) = u(b,a) = 1$

So check $u(a,b) \overset{?}{>} u(b,b)$

$1 > 0$ ✓

So a is ES

Defn change:

$\hat{s} \rightsquigarrow \hat{p}$

pure $\rightsquigarrow$ mixed

Evolution of social convention: driving on L or R

	L	R
L	2,2	0,0
R	0,0	1,1

<< What are potential ES? >>

(L,L) and (R,R) are both NE

Strict, so L is ES

R is ES

Lesson: We can have multiple ES conventions

These need not be equally good.

<< (2,2) "better than" (1,1) >>

symmetric B of S

	a	b
a	0,0	2,1
b	1,2	0,0

<< Nature interpretation: a - aggression, b - non-aggression >>

monomorphic population

There is no symmetric pure-strategy NE in the game

<< So no pure, stable gene mix possible >>

There is a symmetric mixed-strategy NE in the game

polymorphic pop.

$\left[\left(\frac{2}{3},\frac{1}{3}\right),\left(\frac{2}{3},\frac{1}{3}\right)\right]$ is NE

($\frac{2}{3}$: aggressive genes; $\frac{1}{3}$: non-aggressive genes)

mixed eq. cannot be strict, since it is mixed

need to check $u(\hat{p},p') \overset{?}{\geq} u(p',p')$ for all possible mixed mutations p' ✓

Hawk-Dove (strategy names for same species)

	H	D
H	$\frac{V-C}{2}, \frac{V-C}{2}$	V, 0
D	0, V	$\frac{V}{2}, \frac{V}{2}$
	$\hat{p}$	$(1-\hat{p})$

prize = V > 0

costs = C > 0 of fight

(Is D an Ess?

Is (D,D) a NE? ✗ so not ESS

(Is H an ESS?

Is (H,H) a NE? Yes if $\frac{V-C}{2} \geq 0$

case (1) $V > C$ then (H,H) is strict NE

(2) $V = C \Rightarrow \frac{V-C}{2} = 0$

$u(H,H) = u(D,H)$ ...

Check $u(H,D) \stackrel{?}{>} u(D,D)$

$V > \frac{V}{2}$ ✓

shown: if $V \geq C$
the H is an ESS

If $C > V$ we know H is not ESS
D is not ESS

What about $\hat{p}$?

Step one: find a symmetric mixed NE $(\hat{p}, 1-\hat{p})$

$$u(H,\hat{p}) = \hat{p}\left(\frac{V-C}{2}\right) + (1-\hat{p})\,V$$
$$u(D,\hat{p}) = \hat{p}\,[0] + (1-\hat{p})\frac{V}{2}$$
(trust) $\Rightarrow \boxed{\hat{p} = \frac{V}{C}}$

$\left(\frac{V}{C}, 1-\frac{V}{C}\right)$

NOT STRICT
we need to check

$u(\hat{p},p') \stackrel{?}{\geq} u(p',p')$ for all possible p'

Heuristic argument:
Hawkish mutations do poorly against self, die out
Dove-ish " " " " " " " "

In fact: ✓

Lessons: If $V < C$ then ES has $\frac{V}{C}$ Hawks

a) as $V \uparrow$, more Hawks in ESS
as $C \uparrow$, more Doves in ESS

b) payoffs $= \left(1-\frac{V}{C}\right)\left(\frac{V}{2}\right)$

Dove payoff must be same as Hawk payoff

what happens as $C \uparrow$? the payoff $\uparrow$!

c) Identification we can tell what $\frac{V}{C}$ is from data

	S	B	T	
S	1,1	V,0	0,V	$\frac{1}{3}$
B	0,V	1,1	V,0	$\frac{1}{3}$
T	V,0	0,V	1,1	$\frac{1}{3}$
	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	

$1 < V < 2$

The only hope for an ESS is $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

It is NE.
It is weak NE

Check $u(\hat{p},p') \stackrel{?}{\geq} u(p',p')$

Let $p' = S$

$u(\hat{p},S) = \frac{1+V}{3} < 1$

$u(S,S) = 1$ ← ✓ bigger.

example: NO ESS !

What happens?
Cycling around!

Orange lizards - harems
Yellow lizards - sneaky
blue lizards - monogamy