ASTRO 160
(part deux)
BLACK HOLES & RELATIVITY

(note: no problem set this week)

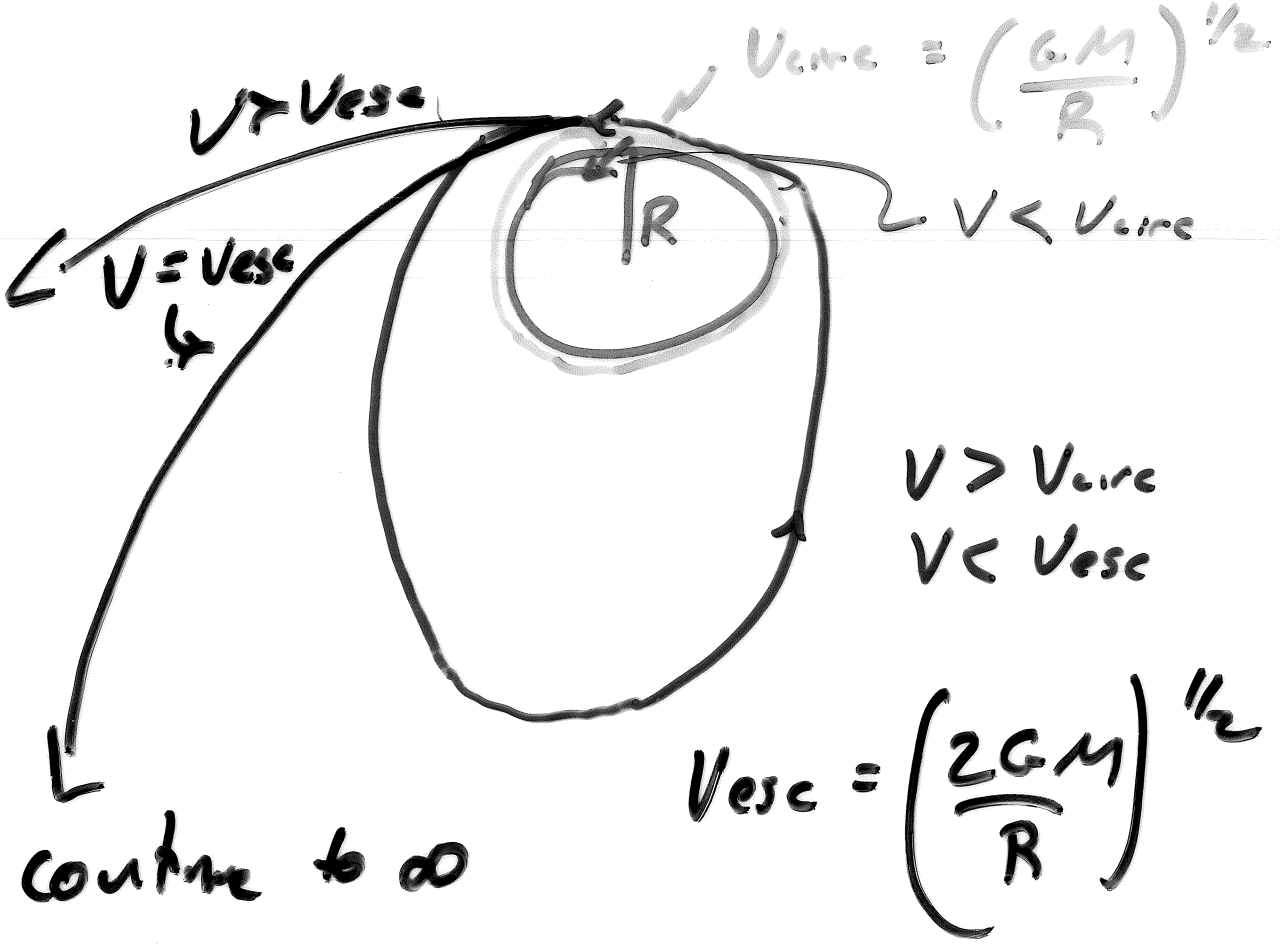
BLACK HOLES
not seen directly
use — orbital dynamics
Doppler shift etc.

context: relativity not Newtonian physics

"escape velocity"

$$V_{esc} = \left(\frac{2GM}{R}\right)^{1/2}$$

speed required to escape grav. field of an object with mass = M radius = R
(if you are on the surface)



V_{esc} of Earth

$$V_{esc} = \left(\frac{2GM}{R}\right)^{1/2} = \left(\frac{2 \times 7 \times 10^{-11} \; 6 \times 10^{24}}{7 \times 10^{6}}\right)^{1/2}$$

$$= \left(\frac{12 \times 10^{13}}{10^{6}}\right)^{1/2} = \left(1.2 \times 10^{8}\right)^{1/2}$$

$\sim 10^{4}$ m/s

~ 10 km/s

V_{esc} of human

$M = 100\,kg$

$R = 1\,m$

$$V_{esc} = \left(\frac{2GM}{R}\right)^{1/2}$$

$$= \left(\frac{2 \times 7 \times 10^{-11} \times 10^{2}}{1}\right)^{1/2}$$

$$= \left(1.4 \times 10^{-8}\right)^{1/2}$$

$$= 1 \times 10^{-4}\ m/s$$

$$< 1\ m/hr$$

Black Hole: $V_{esc} \geq c$

$\hookrightarrow 3 \times 10^{8}\ m/s$

$v_{esc} = c$

$$c = \left(\frac{2GM}{R}\right)^{1/2}$$

$$R_s = \frac{2GM}{c^2}$$

↑ Schwarzschild radius

black hole:

$R < R_s$

FABLE: Michell's "discovery" of black holes

MORAL: importance of result changes with context

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how big is R_s of Sun?

$$R_s = \frac{2GM}{c^2} = \frac{2 \times 7 \times 10^{-11} \; 2 \times 10^{30}}{(3 \times 10^8)^2}$$

$$= \frac{30 \times 10^{19}}{10 \times 10^{16}} = 3 \times 10^3 \text{ m}$$

3 km

Known for $\geq$ 70 yrs that black holes SHOULD exist - endpoints of stellar evolution

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star's evolution determined by two forces

gravity (pulls in)

pressure (push out)

"hydrostatic equilibrium"

P_{out} ↑ ↓ gravity

P_{out} ~~P_{in}~~ = Gravity

$P_{in} - P_{out}$

$P_{inside} > P_{outside}$

"gas pressure"

$PV = nRT$

V → volume

n → # of particle per volume

R → const.

$\frac{n}{V} = \rho \times \text{const.}$

$$P = \text{const} \times \rho \times T$$

inside T, ρ bigger than outside

if only ρ varies

inner regions → higher pressure
→ higher gravity

no balance is possible

⇒ inner parts of star MUST be hotter than outer part

inside of Sun 10^7 degrees
surface of Sun 6×10^3 degrees

heat in center of star flows out (at surface radiates)

reason: an energy source
at core of star
→ replaces lost heat
→ preserves equilibrium

NUCLEAR FUSION

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eventually nuclear fuel
runs out
→ many adventures
→ collapses

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at high density
→ other kinds of pressure
"electron degeneracy pressure"
(fermi pressure)

Stabilizes star at around
radius of Earth

$$\rho = \frac{M}{V} = \frac{2\times10^{30}}{\frac{4}{3}\pi(7\times10^{6})^{3}}$$

$$= \frac{2\times10^{30}}{4\times350\times10^{18}}$$

$$= \frac{2000\times10^{33}}{1400\times10^{18}} = 10^{9}\ \mathrm{kg/m^{3}}$$

million times denser
than water

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such stars called
white dwarf

endpoint of Sun

1930s Chandrasekhar
proves that
$M > 1.4 M_{\odot}$

electron pressure
insufficient

FABLE: Chandra's Limit

MORAL: believe your skill not your intuition

When white dwarf collapses

$e + p \rightarrow n + \nu$ → neutrinos

white star turns into neutrons

NEUTRON STARS
(discovered in 1960s)

$M = 2 M_\odot$ $R \sim 10$ km

S is a billion x smaller than white dwarfs!

$$\frac{R_{ns}}{R_\odot} = \frac{2 G M_{ns}}{c^2} \cdot \frac{1}{2 G M_\odot / c^2}$$

$$= \frac{M_{ns}}{M_\odot}$$

If $R_{S\odot} = 3$ km

R_{S*} = 10 km

with $M = 3 M_\odot$

= radius of neutron star

neutron star with $M > 3 M_\odot$, $R < R_S$ ⇒ black hole

stars with initial mass $> 30 M_\odot$ will end up with $M > 3 M_\odot$ ⟶ collapse ⇒ black holes

⇒ EXPECT THERE ARE MANY BLACK HOLES