SPECIAL RELATIVITY

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$v/c \to 0$ $\quad \gamma \to 1$
(Newtonian)

$v/c \to 1$ $\quad \gamma \to \infty$
(Relativity)

example: $m = \gamma m_0$

m ↑ rel. mass

m_0 ↑ rest mass

$v \to c$ $\quad m \to \infty$
no more acceleration

$F = m \cdot a$

HENCE cannot go faster than c

photon has zero m_0

"post-Newtonian" approx.

v^2/c^2 is small but not zero

$$(1+\varepsilon)^n = 1 + n\varepsilon + \ldots$$

ε something small, $\varepsilon \ll 1$

$\ldots$ negligible

$$\Rightarrow (1+\varepsilon)^n \approx 1 + n\varepsilon \quad \text{if } \varepsilon \ll 1.$$

Newtonian

"post-Newtonian" approx

Earth's orbit: $v = 3\times10^4$ m/s

$c = 3\times10^8$ m/s

$$\frac{v^2}{c^2} = \left(\frac{3\times10^4}{3\times10^8}\right)^2 = 10^{-8}$$

$$\gamma = \frac{1}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} = \left(1 - v^2/c^2\right)^{-1/2}$$

$\uparrow$ ignored

$$\approx 1 + \frac{1}{2}v^2/c^2 + \ldots - \left(\frac{v}{c}\right)^2$$

$$m = m_0 \gamma$$

$$= m_0\left(1 + \tfrac{1}{2} v^2/c^2\right)$$

$$= m_0 + \underbrace{\tfrac{1}{2} m_0 v^2}\ \frac{1}{c^2}$$

m_0: Newtonian

$\tfrac{1}{2} m_0 v^2$: kinetic energy

$$m = m_0 + \frac{K.E.}{c^2}$$

$\frac{K.E.}{c^2}$: mass equivalent kinetic energy

$$\boxed{\frac{E}{c^2} = m}$$

$m = m_0\gamma$:

- $\frac{v^2}{c^2} \to 0$ — Newtonian
- $v^2/c^2 \to 1$ — light is a speed limit
- $v^2/c^2 < 1$ — $E = mc^2$

What is mass?

$F = ma$

↓

higher mass requires greater force to accelerate

"inertial mass"

$t = \gamma t_0$ time dilation

$L = \frac{L_0}{\gamma}$ length contraction

"Lorentz transformations"

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$v/c \to 0 \quad \gamma \to 1$$

$M_{inertial} = M_{intrinsic}$

↑ rest mass

$$\frac{1}{(1 - v^2/c^2)^{1/2}} \approx 1 + \frac{1}{2} v^2/c^2 + \dots$$

↑ rest mass

↑ $\frac{\text{Kinetic } E}{c^2}$

Force : ability to do work

$$E = mc^2$$

total energy (points to E)

inertial mass (points to m)

$$m = M_0 + \frac{K.E.}{c^2}$$

kinetic energy (points to K.E.)

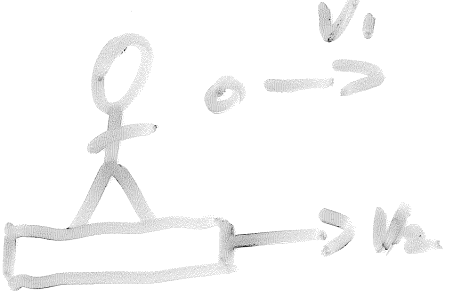
$M_0 \equiv \dfrac{\text{rest energy}}{c^2}$

$$E = c^2 m_0 + K.E$$

$$t = \gamma t_0$$

$$\frac{1}{\sqrt{1 - v^2/c^2}} \Rightarrow \text{imaginary}$$

The speed of light is same ~~all~~ for all observers.



Newton

$\Rightarrow V_{TOT} = V_1 + V_2$

$\Rightarrow V_{TOT} = c$

NOT $c + V_2$

Hmmm... velocity is $\frac{\text{space}}{\text{time}}$

space messed up close to c.