

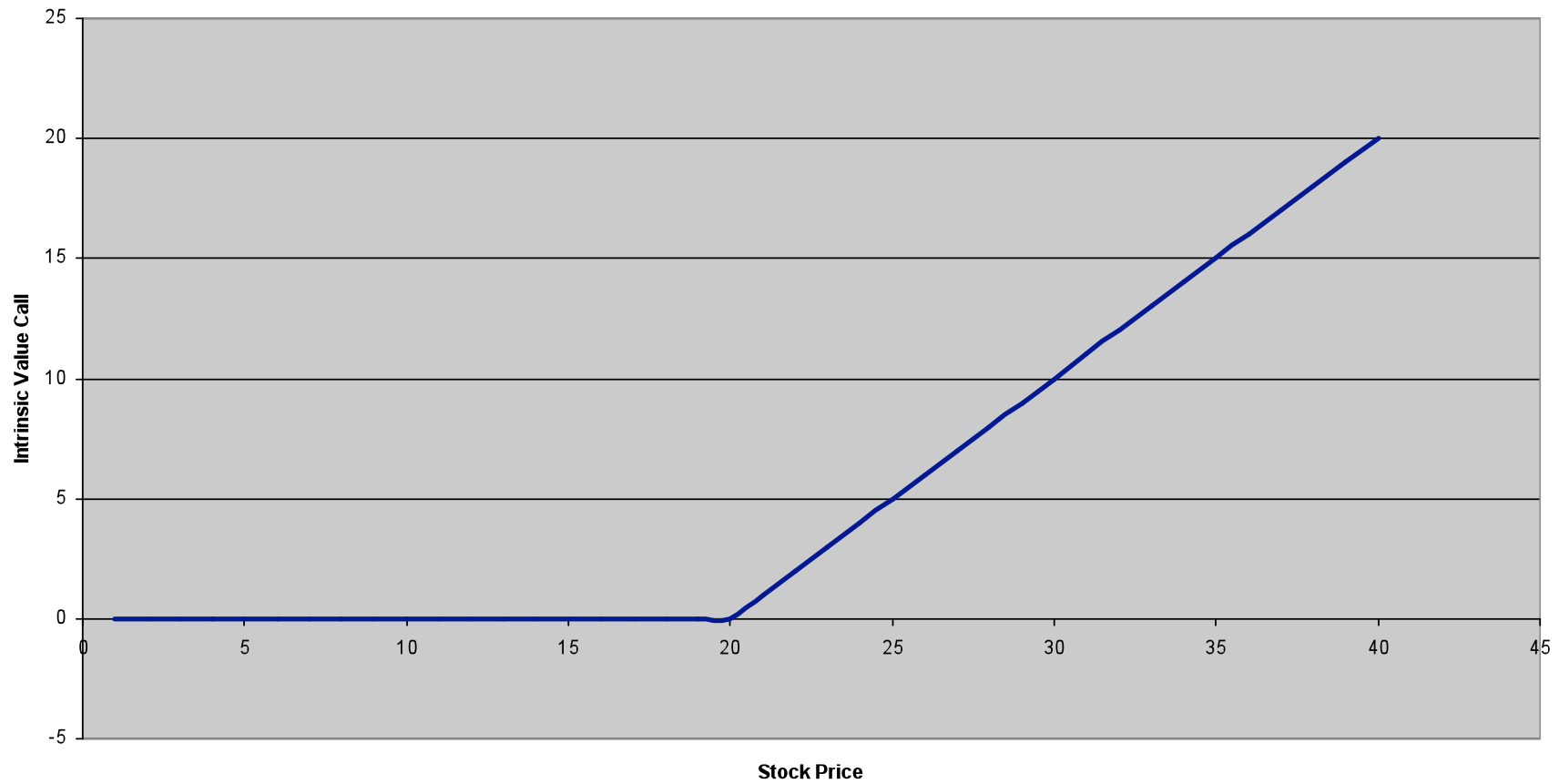
Lecture 23: Options Markets

Economics 252, Spring 2008

Prof. Robert Shiller, Yale University

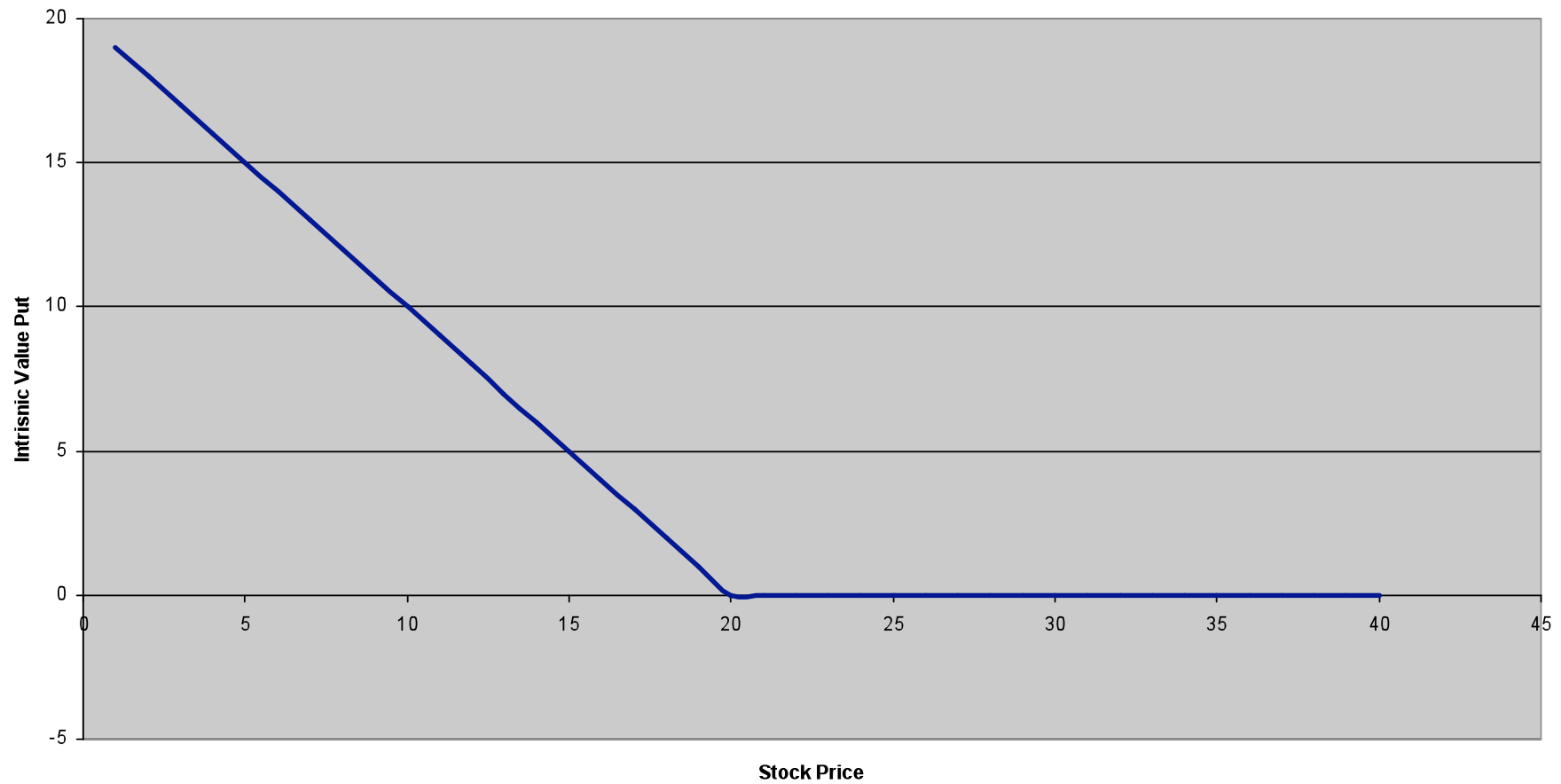
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Exercise Price = 20



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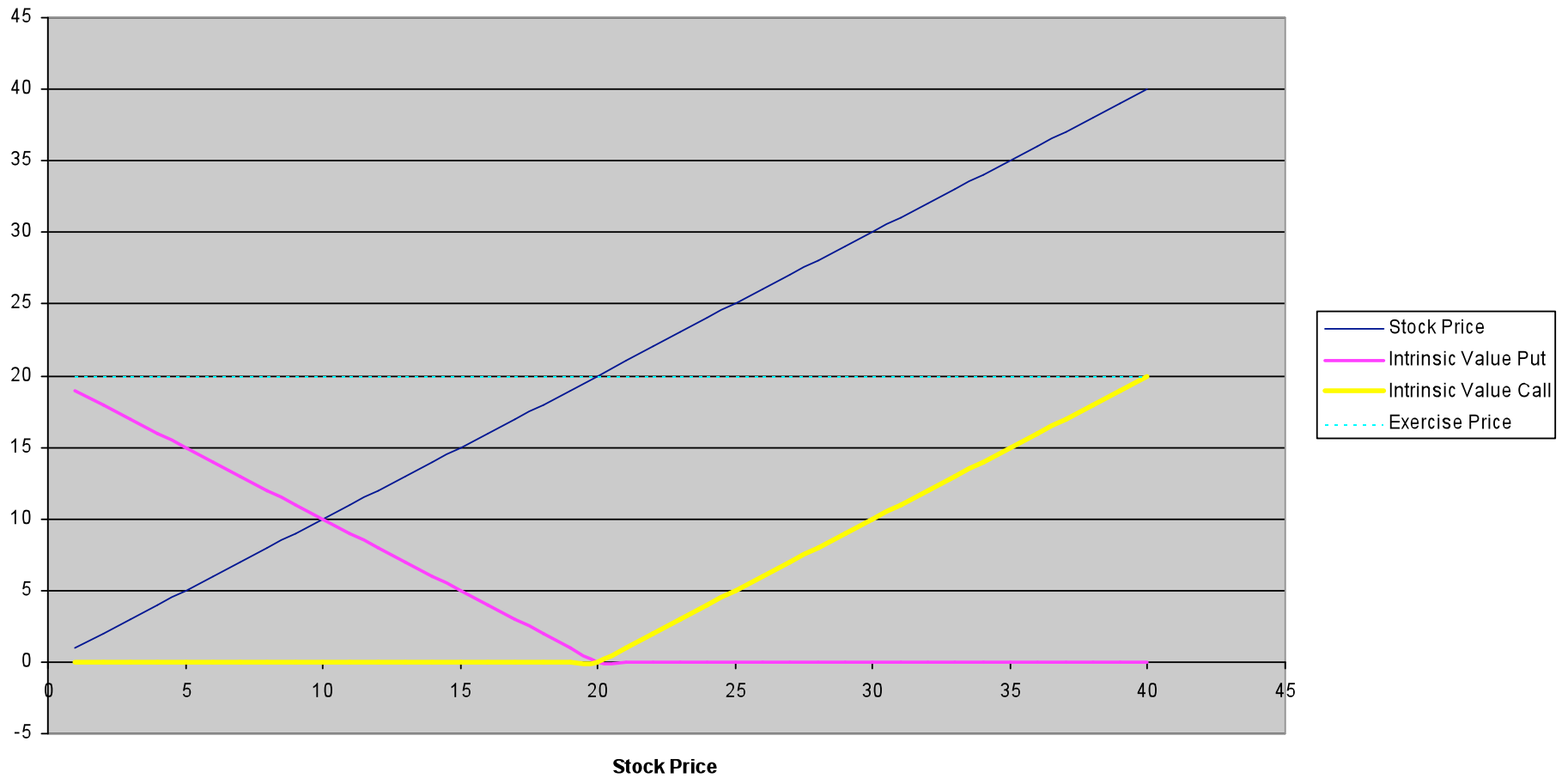


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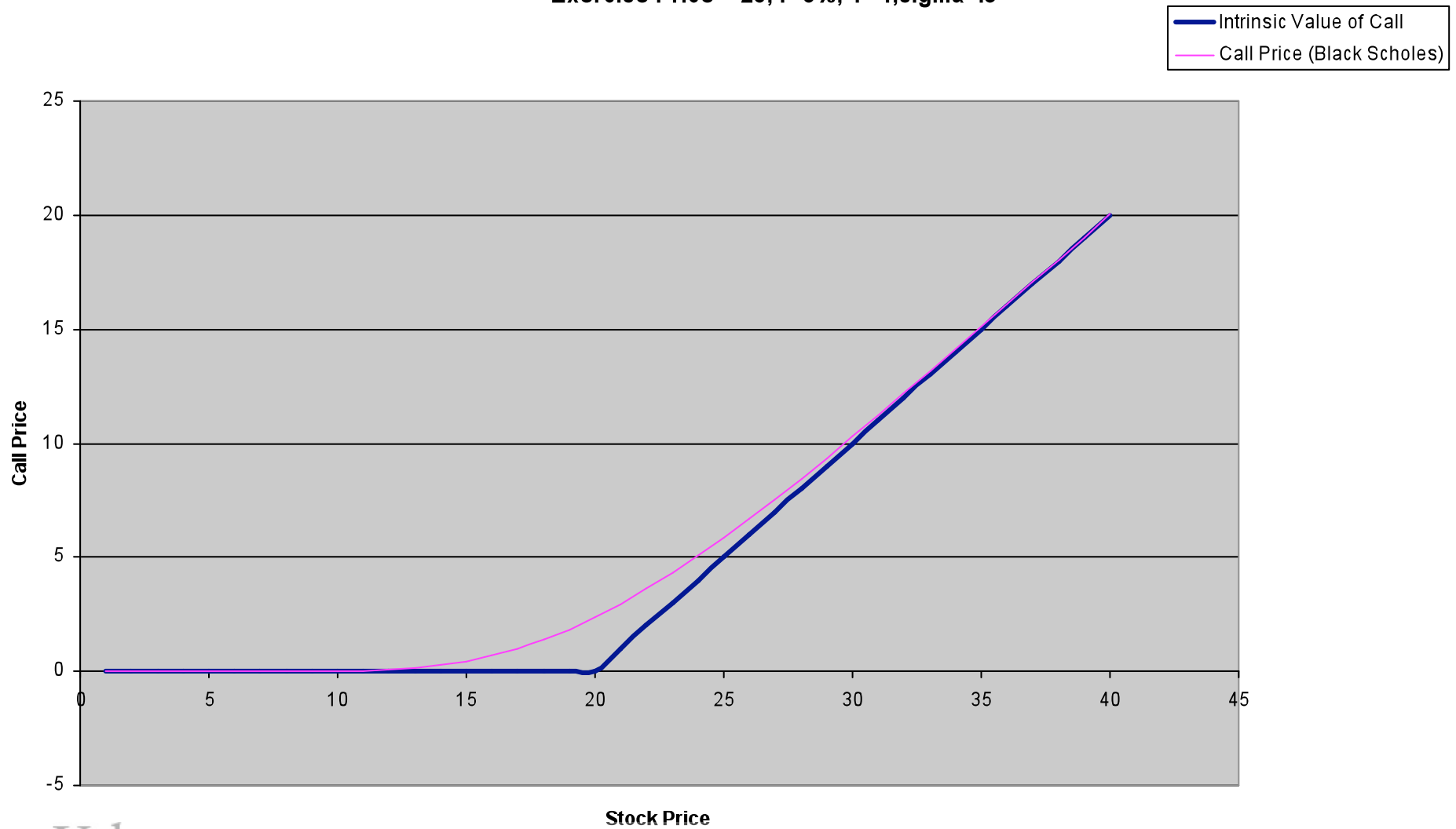
Put-Call Parity Relation

- Put option price – call option price = present value of strike price + present value of dividends – price of stock
- For European options, this formula must hold (up to small deviations due to transactions costs), otherwise there would be arbitrage profit opportunities

Put Call Parity Relation Derivation



Exercise Price = 20, $r=5\%$, $T=1$, $\sigma=.3$



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Binomial Option Pricing

- S = current stock price
- $u = 1 + \text{fraction of change in stock price if price goes up}$
- $d = 1 + \text{fraction of change in stock price if price goes down}$
- r = risk-free interest rate

Binomial Option Pricing, Cont.

- C = current price of call option
- C_u = value of call next period if price is up
- C_d = value of call next period if price is down
- E = strike price of option
- H = hedge ratio, number of shares purchased per call sold

Hedging by writing calls

- Investor writes one call and buys H shares of underlying stock
- If price goes up, will be worth $uHS - C_u$
- If price goes down, worth $dHS - C_d$
- For what H are these two the same?

$$H = \frac{C_u - C_d}{(u - d)S}$$

Binomial Option Pricing Formula

- One invested $HS-C$ to achieve riskless return, hence the return must equal $(1+r)$
 $(HS-C)$
- $(1+r)(HS-C) = uHS-C_u = dHS-C_d$
- Subst for H , then solve for C

$$C = \left(\frac{1+r-d}{u-d}\right)\left(\frac{C_u}{1+r}\right) + \left(\frac{u-1-r}{u-d}\right)\left(\frac{C_d}{1+r}\right)$$

Black-Scholes Option Pricing

Call T the time to exercise, σ^2 the variance of one-period price change (as fraction) and $N(x)$ the standard cumulative normal distribution function (sigmoid curve, integral of normal bell-shaped curve) =normdist(x,0,1,1) Excel (x, mean,standard_dev, 0 for density, 1 for cum.)

Black-Scholes Formula

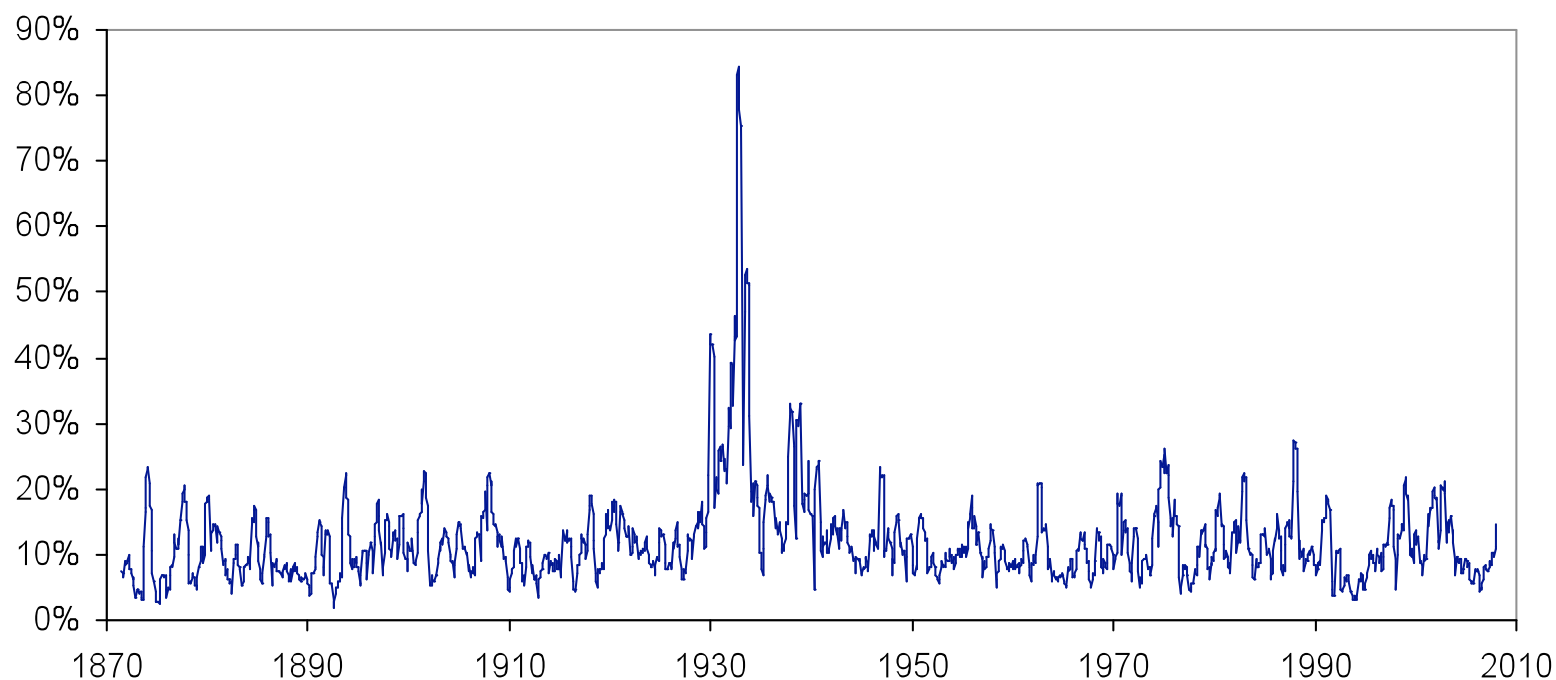
$$C = SN(d_1) - EN(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{E}\right) + rT + \sigma^2 T / 2}{\sigma \sqrt{T}}$$

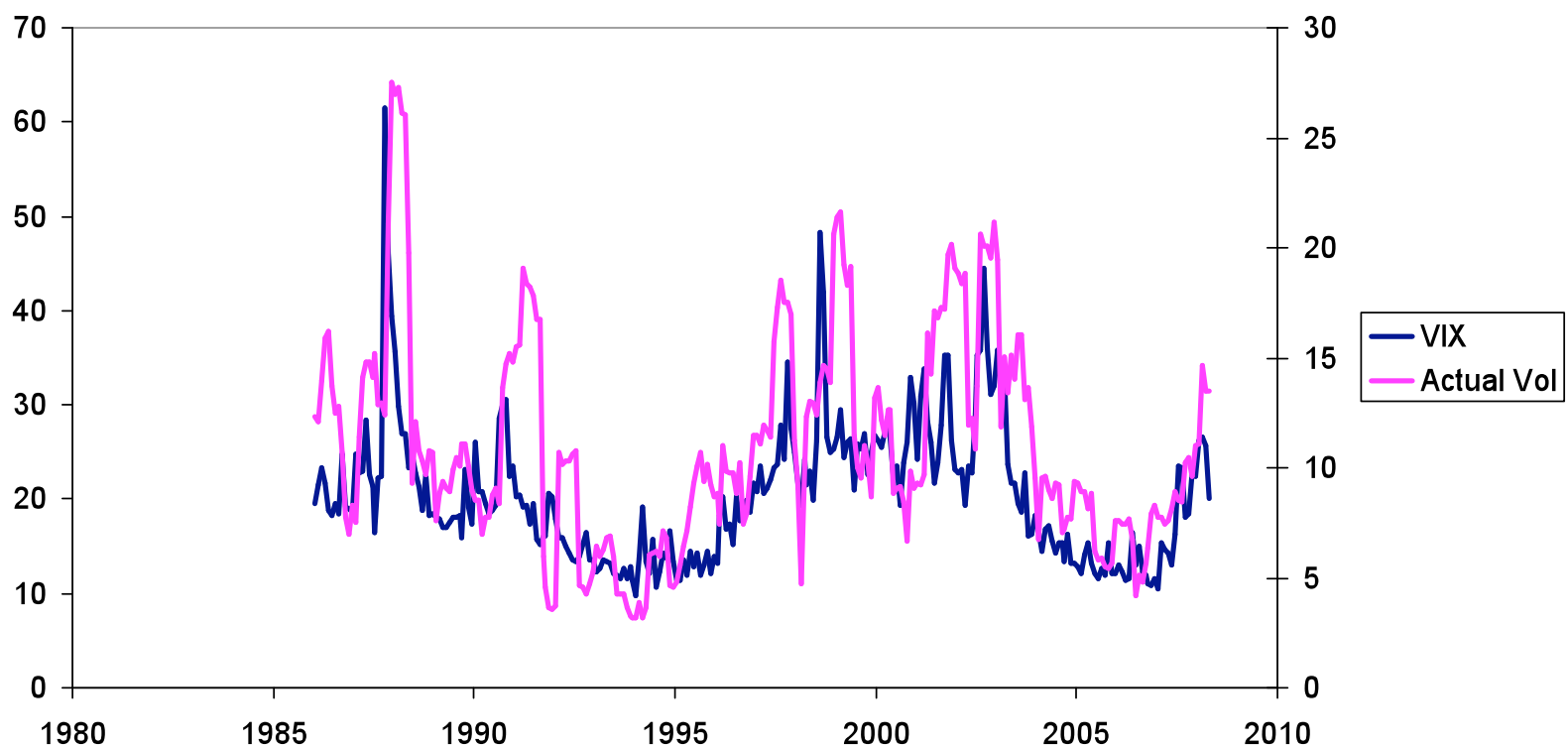
$$d_2 = \frac{\ln\left(\frac{S}{E}\right) + rT - \sigma^2 T / 2}{\sigma \sqrt{T}}$$

Actual S&P500 Volatility Monthly July 1871- April 2008



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Implied and Actual Volatility Monthly Jan 1986-April 2008



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