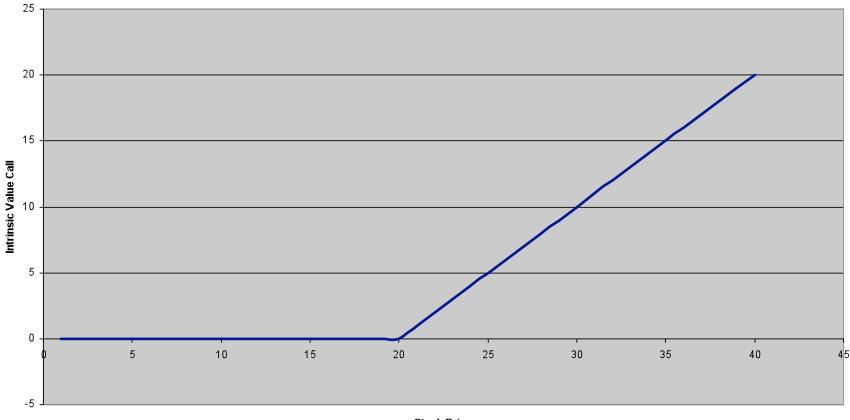
### Lecture 23: Options Markets

#### Economics 252, Spring 2008 Prof. Robert Shiller, Yale University



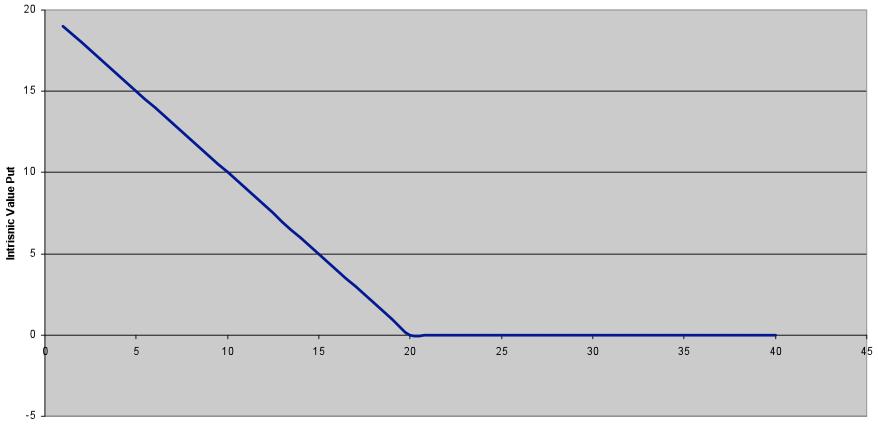
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#### Exercise Price = 20

Stock Price

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Exercise Price = 20

**Stock Price** 

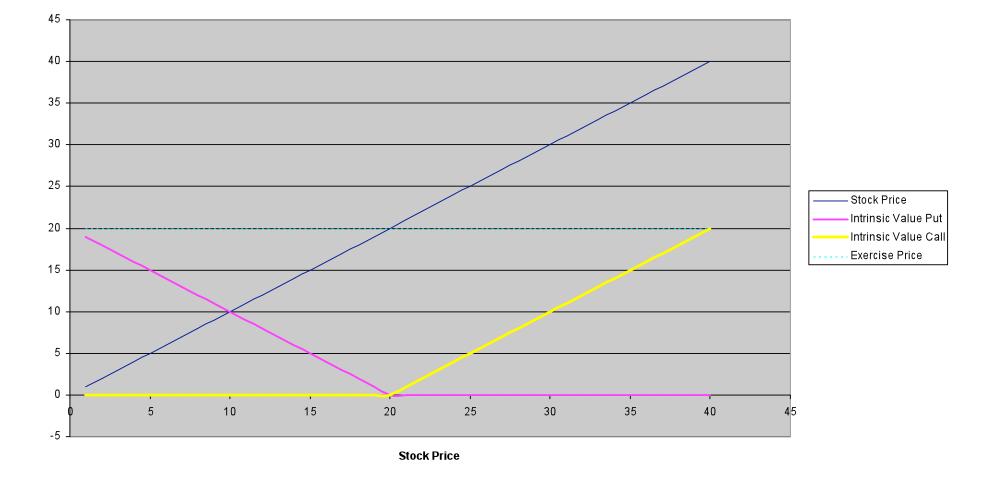
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## Put-Call Parity Relation

- Put option price call option price = present value of strike price + present value of dividends – price of stock
- For European options, this formula must hold (up to small deviations due to transactions costs), otherwise there would be arbitrage profit opportunities

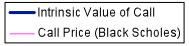


#### Put Call Parity Relation Derivation



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Exercise Price = 20, r=5%, T=1,sigma=.3





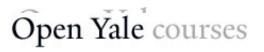
# **Binomial Option Pricing**

- S = current stock price
- *u* = 1+fraction of change in stock price if price goes up
- d = 1+fraction of change in stock price if price goes down
- *r* = risk-free interest rate



# Binomial Option Pricing, Cont.

- C =current price of call option
- $C_u$  = value of call next period if price is up
- $C_d$  = value of call next period if price is down
- E =strike price of option
- *H* = hedge ratio, number of shares purchased per call sold



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# Hedging by writing calls

- Investor writes one call and buys *H* shares of underlying stock
- If price goes up, will be worth  $uHS-C_u$
- If price goes down, worth  $dHS-C_d$
- For what *H* are these two the same?

$$H = \frac{C_u - C_d}{(u - d)S}$$



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## **Binomial Option Pricing Formula**

- One invested *HS-C* to achieve riskless return, hence the return must equal (1+r) (*HS-C*)
- $(1+r)(HS-C)=uHS-C_u=dHS-C_d$
- Subst for *H*, then solve for *C*

$$C = (\frac{1+r-d}{u-d})(\frac{C_u}{1+r}) + (\frac{u-1-r}{u-d})(\frac{C_d}{1+r})$$



# **Black-Scholes Option Pricing**

Call *T* the time to exercise,  $\sigma^2$  the variance of oneperiod price change (as fraction) and N(x) the standard cumulative normal distribution function (sigmoid curve, integral of normal bell-shaped curve) =normdist(x,0,1,1) Excel (x, mean,standard\_dev, 0 for density, 1 for cum.)

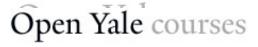


#### Black-Scholes Formula

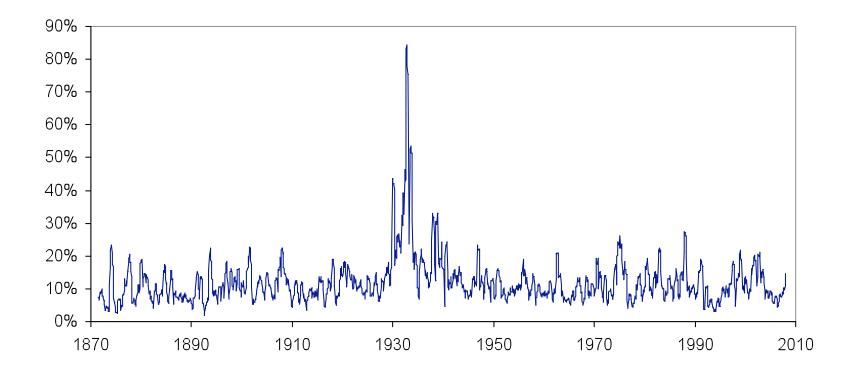
$$C = SN(d_1) - EN(d_2)$$

where

$$d_{1} = \frac{\ln(\frac{S}{E}) + rT + \sigma^{2}T/2}{\sigma \sqrt{T}}$$
$$d_{2} = \frac{\ln(\frac{S}{E}) + rT - \sigma^{2}T/2}{\sigma \sqrt{T}}$$

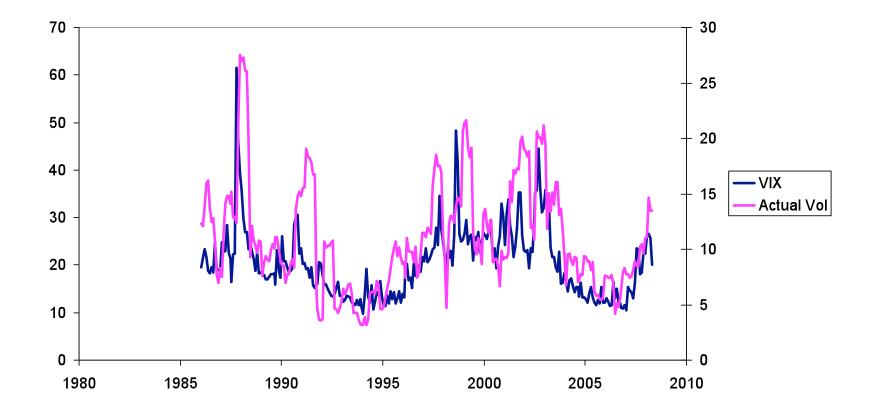


### Actual S&P500 Volatility Monthly July1871- April 2008



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#### Implied and Actual Volatility Monthly Jan 1986-April 2008



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