Lecture 23: Options Markets

Economics 252, Spring 2008
Prof. Robert Shiller, Yale University
Exercise Price = 20

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Intrinsic Value Call

0   5   10   15   20   25   30   35   40   45
0   5   10   15   20   25   30   35   40   45
```

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Stock Price

-5   0   5   10   15   20   25   30   35   40
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Put-Call Parity Relation

- Put option price – call option price = present value of strike price + present value of dividends – price of stock
- For European options, this formula must hold (up to small deviations due to transactions costs), otherwise there would be arbitrage profit opportunities
Binomial Option Pricing

• $S =$ current stock price
• $u = 1+$fraction of change in stock price if price goes up
• $d = 1+$fraction of change in stock price if price goes down
• $r =$ risk-free interest rate
Binomial Option Pricing, Cont.

- $C =$ current price of call option
- $C_u =$ value of call next period if price is up
- $C_d =$ value of call next period if price is down
- $E =$ strike price of option
- $H =$ hedge ratio, number of shares purchased per call sold
Hedging by writing calls

• Investor writes one call and buys $H$ shares of underlying stock
• If price goes up, will be worth $uHS - C_u$
• If price goes down, worth $dHS - C_d$
• For what $H$ are these two the same?

$$H = \frac{C_u - C_d}{(u - d)S}$$
Binomial Option Pricing Formula

• One invested $HS-C$ to achieve riskless return, hence the return must equal $(1+r)(HS-C)$

• $(1+r)(HS-C)=uHS-C_u=dHS-C_d$

• Subst for $H$, then solve for $C$

$$C = \left( \frac{1+r-d}{u-d} \right) \left( \frac{C_u}{1+r} \right) + \left( \frac{u-1-r}{u-d} \right) \left( \frac{C_d}{1+r} \right)$$
Black-Scholes Option Pricing

Call $T$ the time to exercise, $\sigma^2$ the variance of one-period price change (as fraction) and $N(x)$ the standard cumulative normal distribution function (sigmoid curve, integral of normal bell-shaped curve) =normdist(x,0,1,1) Excel (x, mean,standard_dev, 0 for density, 1 for cum.)
Black-Scholes Formula

\[ C = S \text{N}(d_1) - E \text{N}(d_2) \]

where

\[ d_1 = \frac{\ln \left( \frac{S}{E} \right) + rT + \sigma^2 T / 2}{\sigma \sqrt{T}} \]

\[ d_2 = \frac{\ln \left( \frac{S}{E} \right) + rT - \sigma^2 T / 2}{\sigma \sqrt{T}} \]
Actual S&P500 Volatility
Monthly July 1871 - April 2008
Implied and Actual Volatility
Monthly Jan 1986-April 2008