# Lecture 2 : Multiplication Rule and Insurance

- Probability P, 0<P<1
- Multiplication rule for independent events: Prob(A and B) = Prob(A) Prob(B)
- Probability of *n* independent accidents =  $P^n$
- Probability of *x* accidents in *n* policies (Binomial Distributon):

$$f(x) = P^{x}(1-P)^{(n-x)}n!/(x!(n-x)!)$$

# Expected Value, Mean, Average

$$E(x) = \mu_x = \sum_{i=1}^{\infty} prob(x = x_i)x_i$$

$$E(x) = \mu_x = \int_{-\infty}^{\infty} f(x)xdx$$

$$\frac{1}{x} = \sum_{i=1}^{n} x_i / n$$

$$G(x) = (\prod_{i=1}^{n} x_i)^{1/n}$$

#### Open Yale courses

Yale University 2012. Most of the lectures and course material within Open Yale Courses are licensed under a Creative Commons Attribution-Noncommercial-Share Alike 3.0 license. Unless explicitly set forth in the applicable Credits section of a lecture, third-party content is not covered under the Creative Commons license. Please consult the Open Yale Courses Terms of Use for limitations and further explanations on the application of the Creative Commons license.

## Variance and Standard Deviation

- Variance (M<sup>2</sup>)is a measure of dispersion
- Standard deviation (x) is square root of variance

$$var(x) = \sum_{i=1}^{n} prob(x = x_i)(x_i - \mu_x)^2$$

$$s_x^2 = \sum_{i=1}^{n} (x_i - \overline{x})^2 / n$$

## Covariance

 A Measure of how much two variables move together

$$cov(x, y) = \sum_{i=1}^{n} (x - \overline{x})(y - \overline{y})/n$$

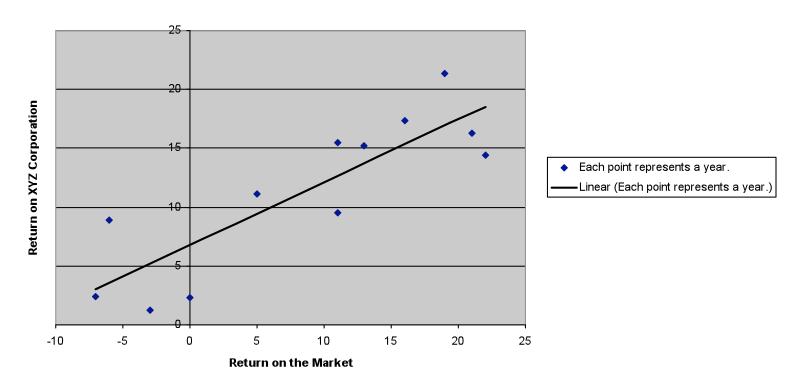
## Correlation

- A scaled measure of how much two variables move together
- -1. W

$$\rho = \operatorname{cov}(x, y) / (s_x s_y)$$

## Regression, Beta=.5, corr=.93

#### Return XYZ Corporation against Market 1990-2001

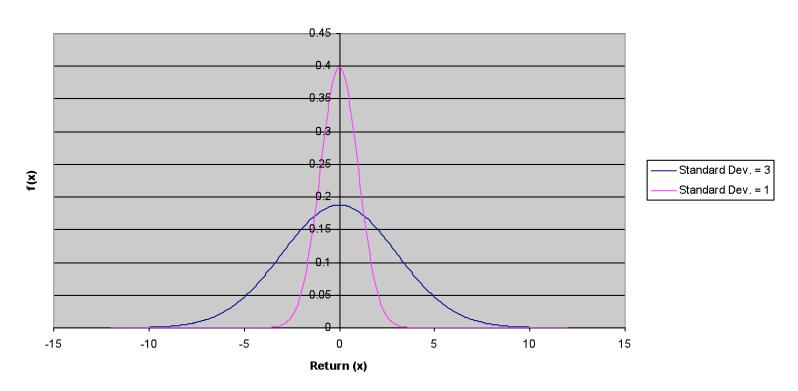


### Open Yale courses

Yale University 2012. Most of the lectures and course material within Open Yale Courses are licensed under a Creative Commons Attribution-Noncommercial-Share Alike 3.0 license. Unless explicitly set forth in the applicable Credits section of a lecture, third-party content is not covered under the Creative Commons license. Please consult the Open Yale Courses Terms of Use for limitations and further explanations on the application of the Creative Commons license.

## Normal Distribution

#### Normal Distribution with Zero Mean

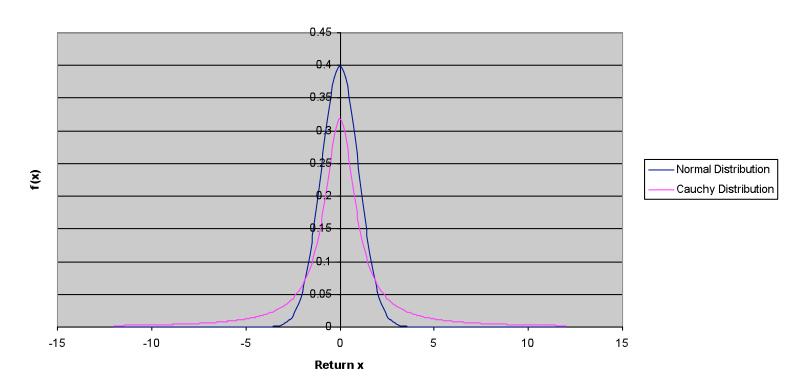


## Open Yale courses

Yale University 2012. Most of the lectures and course material within Open Yale Courses are licensed under a Creative Commons Attribution-Noncommercial-Share Alike 3.0 license. Unless explicitly set forth in the applicable Credits section of a lecture, third-party content is not covered under the Creative Commons license. Please consult the Open Yale Courses Terms of Use for limitations and further explanations on the application of the Creative Commons license.

## Normal Versus Fat-Tailed

#### **Normal Versus Fat Tailed Distributions**



## Open Yale courses

© Yale University 2012. Most of the lectures and course material within Open Yale Courses are licensed under a Creative Commons Attribution-Noncommercial-Share Alike 3.0 license. Unless explicitly set forth in the applicable Credits section of a lecture, third-party content is not covered under the Creative Commons license. Please consult the Open Yale Courses Terms of Use for limitations and further explanations on the application of the Creative Commons license.

## Present Discounted Value (PDV)

- PDV of a dollar in one year = 1/(1+r)
- PDV of a dollar in *n* years =  $1/(1+r)^n$
- PDV of a stream of payments  $x_1,...,x_n$

$$PDV = \sum_{t=1}^{T} x_t / (1+r)^t$$

# Consol and Annuity Formulas

- Consol pays constant quantity x forever
- Growing consol pays  $x(1+g)^{(t-1)}$  in t
- Annuity pays x from time 1 to T

  Consol PDV = x/rGrowing Consol PDV = x/(r-g)Annuity PDV =  $x^{\frac{1-1}{(1+r)^T}}$