Lecture 2 :Multiplication Rule and Insurance

- Probability P, $0<P<1$
- Multiplication rule for independent events: Prob(A and B) = Prob(A) × Prob(B)
- Probability of n independent accidents = P^n
- Probability of x accidents in n policies (Binomial Distributon):

$$f(x) = P^x (1-P)^{(n-x)} n!/(x!(n-x)!)$$

Expected Value, Mean, Average

$$E(x) = \mu_x = \sum_{i=1}^{\infty} prob(x = x_i)x_i$$

$$E(x) = \mu_x = \int_{-\infty}^{\infty} f(x)x\,dx$$

$$\bar{x} = \sum_{i=1}^{n} x_i / n$$

$$\mathrm{G}(x) = \left(\prod_{i=1}^{n} x_i\right)^{1/n}$$

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Variance and Standard Deviation

- Variance (σ^2)is a measure of dispersion
- Standard deviation σ is square root of variance

$$\text{var}(x) = \sum_{i=1}^{n} \text{prob}(x = x_i)(x_i - \mu_x)^2$$

$$s_x^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 / n$$

Covariance

- A Measure of how much two variables move together

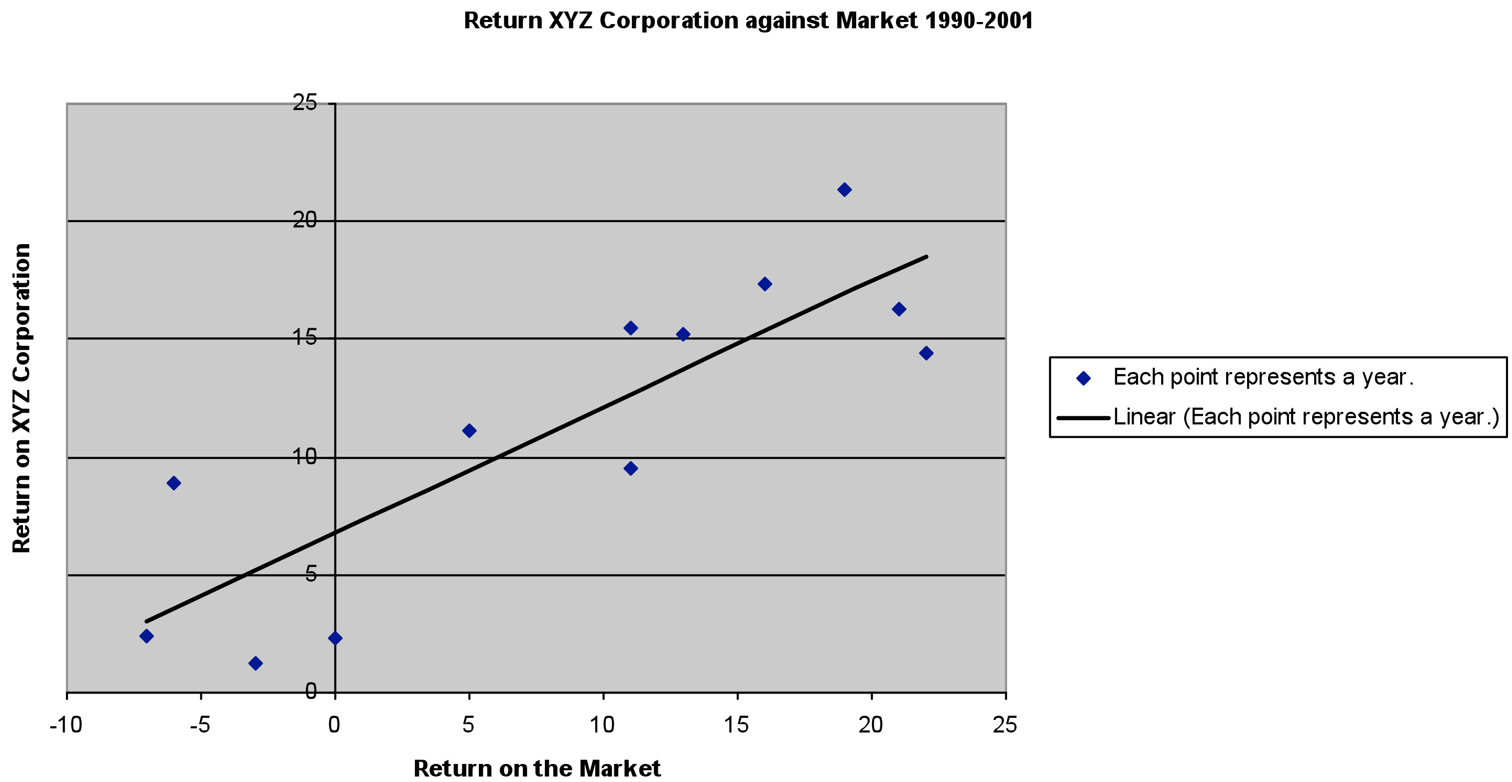
$$\mathrm{cov}(x, y) = \sum_{i=1}^{n} (x - \overline{x})(y - \overline{y}) / n$$

Correlation

- A scaled measure of how much two variables move together
- -1 1

$$\rho = \mathrm{cov}(x, y)/(s_x s_y)$$

Regression, Beta=.5, corr=.93



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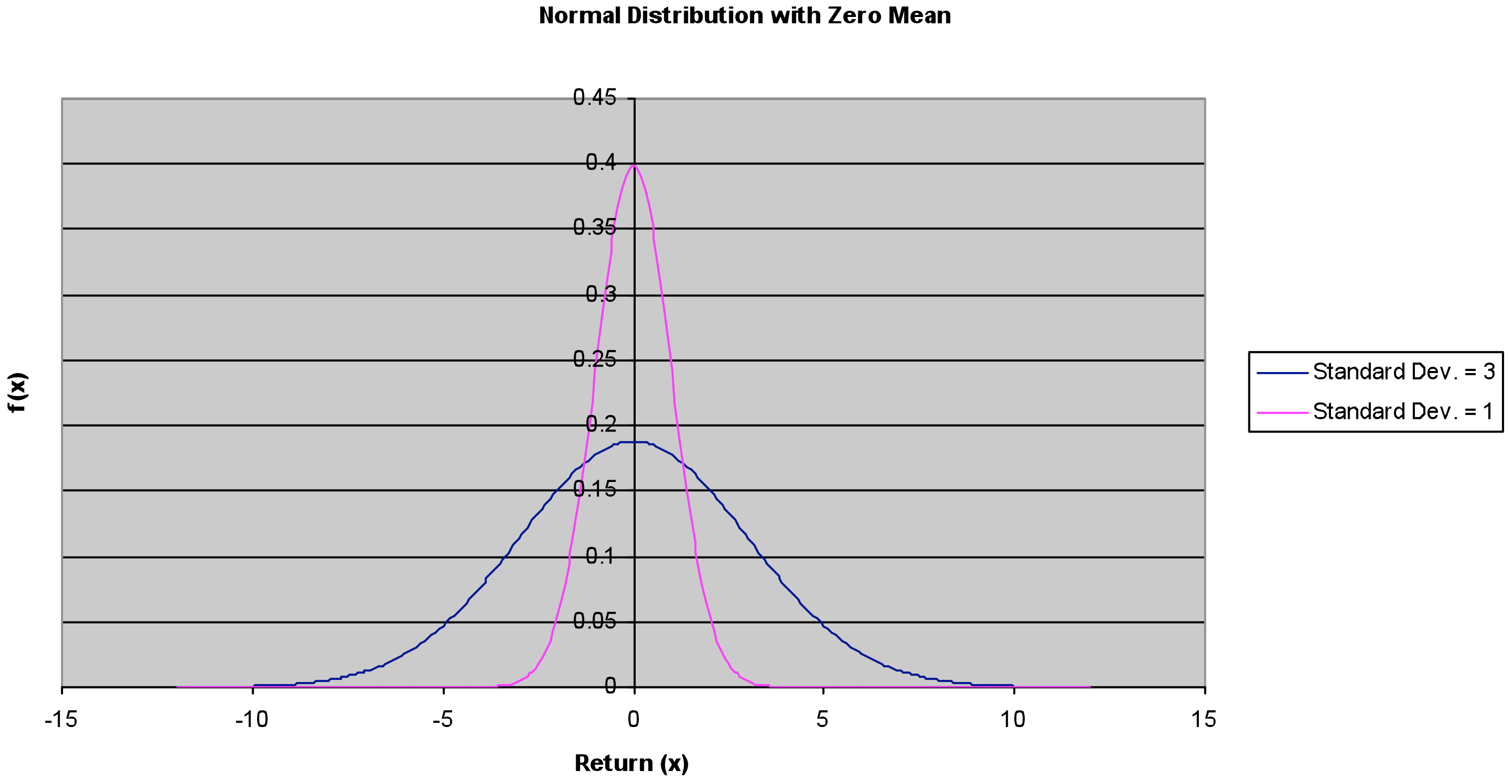
Normal Distribution



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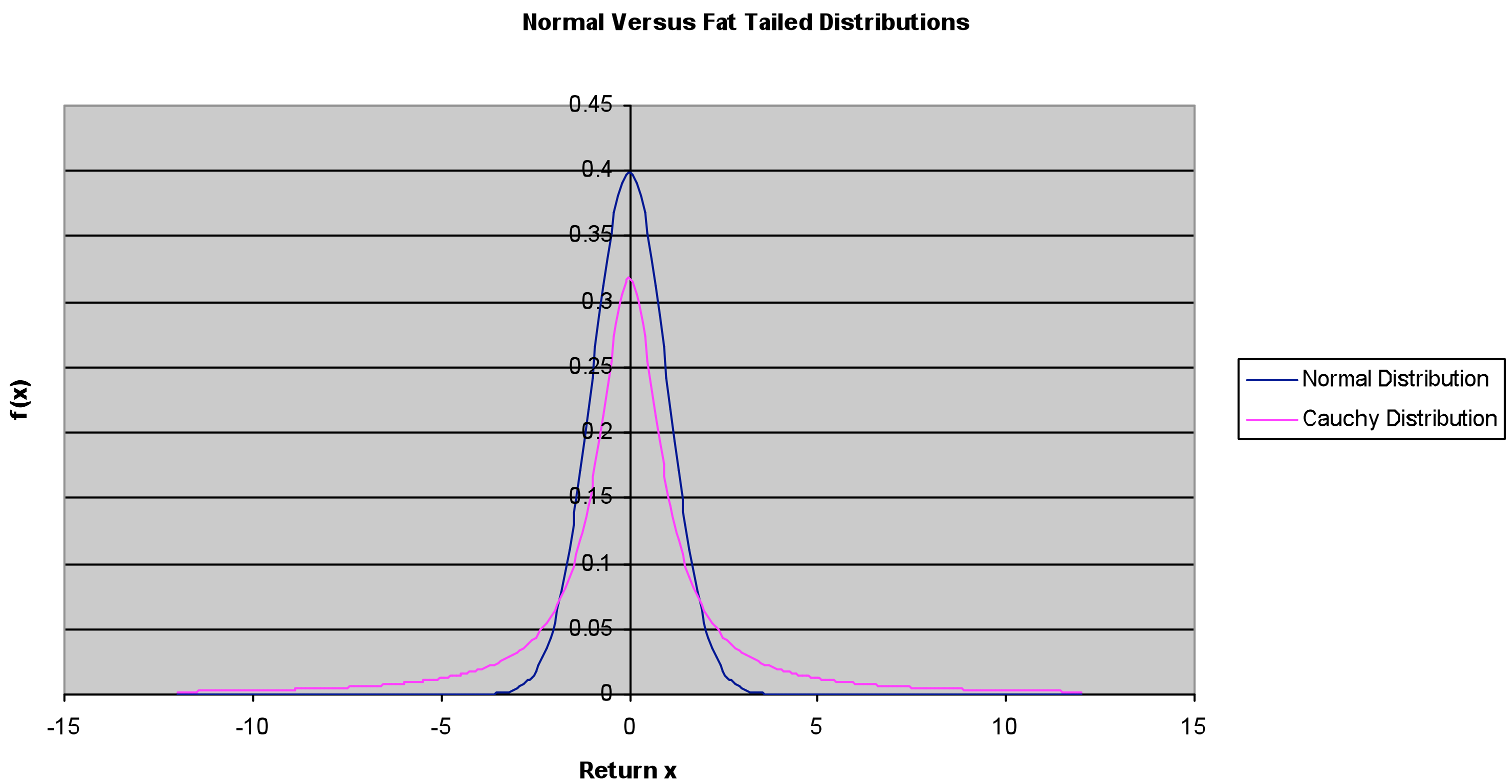
Normal Versus Fat-Tailed



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Present Discounted Value (PDV)

- PDV of a dollar in one year = $1/(1+r)$
- PDV of a dollar in *n* years = $1/(1+r)^n$
- PDV of a stream of payments $x_1,..,x_n$

$$\text{PDV} = \sum_{t=1}^{T} x_t /(1+r)^t$$

Consol and Annuity Formulas

- Consol pays constant quantity x forever
- Growing consol pays $x(1+g)$^$(t$-1) in t
- Annuity pays x from time 1 to T

$$\text{Consol PDV} = x/r$$

$$\text{Growing Consol PDV} = x/(r-g)$$

$$\text{Annuity PDV} = x\frac{1-1/(1+r)^T}{r}$$