

Lecture 2 : Multiplication Rule and Insurance

- Probability P , $0 < P < 1$
- Multiplication rule for independent events:
 $\text{Prob}(A \text{ and } B) = \text{Prob}(A) \text{Prob}(B)$
- Probability of n independent accidents $= P^n$
- Probability of x accidents in n policies
(Binomial Distribution):

$$f(x) = P^x (1 - P)^{(n-x)} n! / (x!(n-x)!)$$

Expected Value, Mean, Average

$$E(x) = \mu_x = \sum_{i=1}^{\infty} \text{prob}(x = x_i) x_i$$

$$E(x) = \mu_x = \int_{-\infty}^{\infty} f(x) x dx$$

$$\bar{x} = \sum_{i=1}^n x_i / n$$

$$G(x) = \left(\prod_{i=1}^n x_i \right)^{1/n}$$

Variance and Standard Deviation

- Variance (σ^2) is a measure of dispersion
- Standard deviation σ is square root of variance

$$\text{var}(x) = \sum_{i=1}^n \text{prob}(x = x_i)(x_i - \mu_x)^2$$

$$s_x^2 = \sum_{i=1}^n (x_i - \bar{x})^2 / n$$

Covariance

- A Measure of how much two variables move together

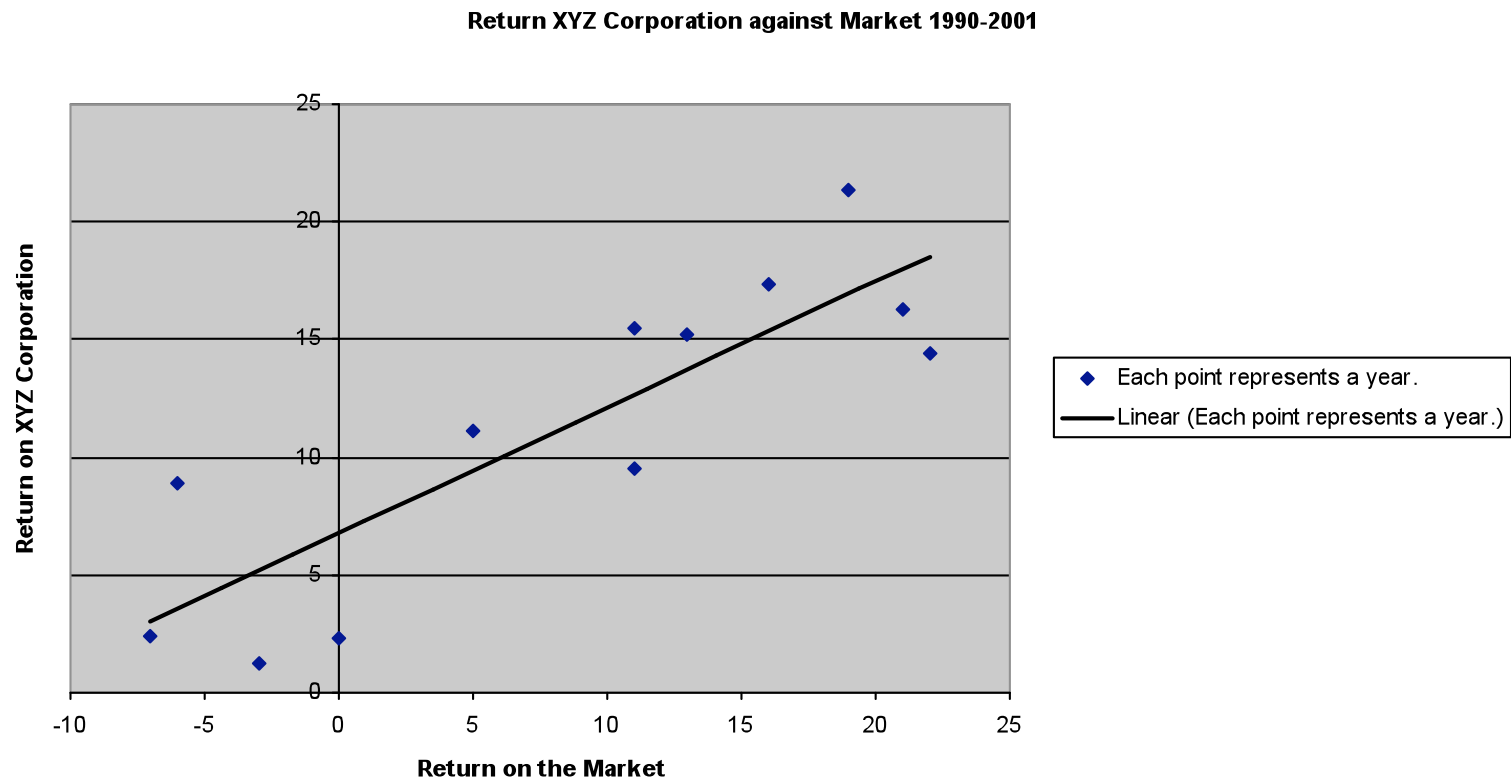
$$\text{cov}(x, y) = \sum_{i=1}^n (x - \bar{x})(y - \bar{y}) / n$$

Correlation

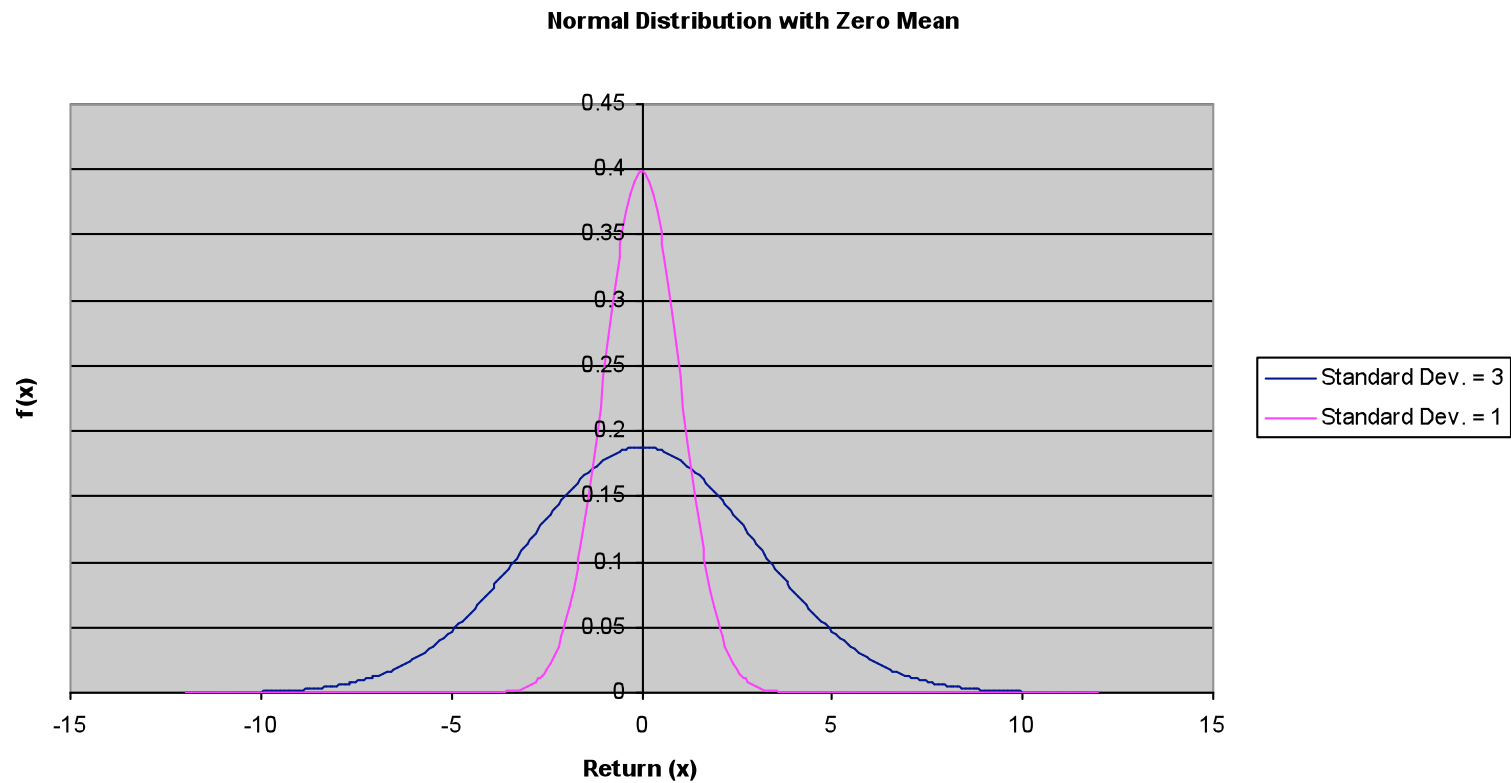
- A scaled measure of how much two variables move together
- $-1 \leq \rho \leq 1$

$$\rho = \text{cov}(x, y) / (s_x s_y)$$

Regression, Beta=.5, corr=.93



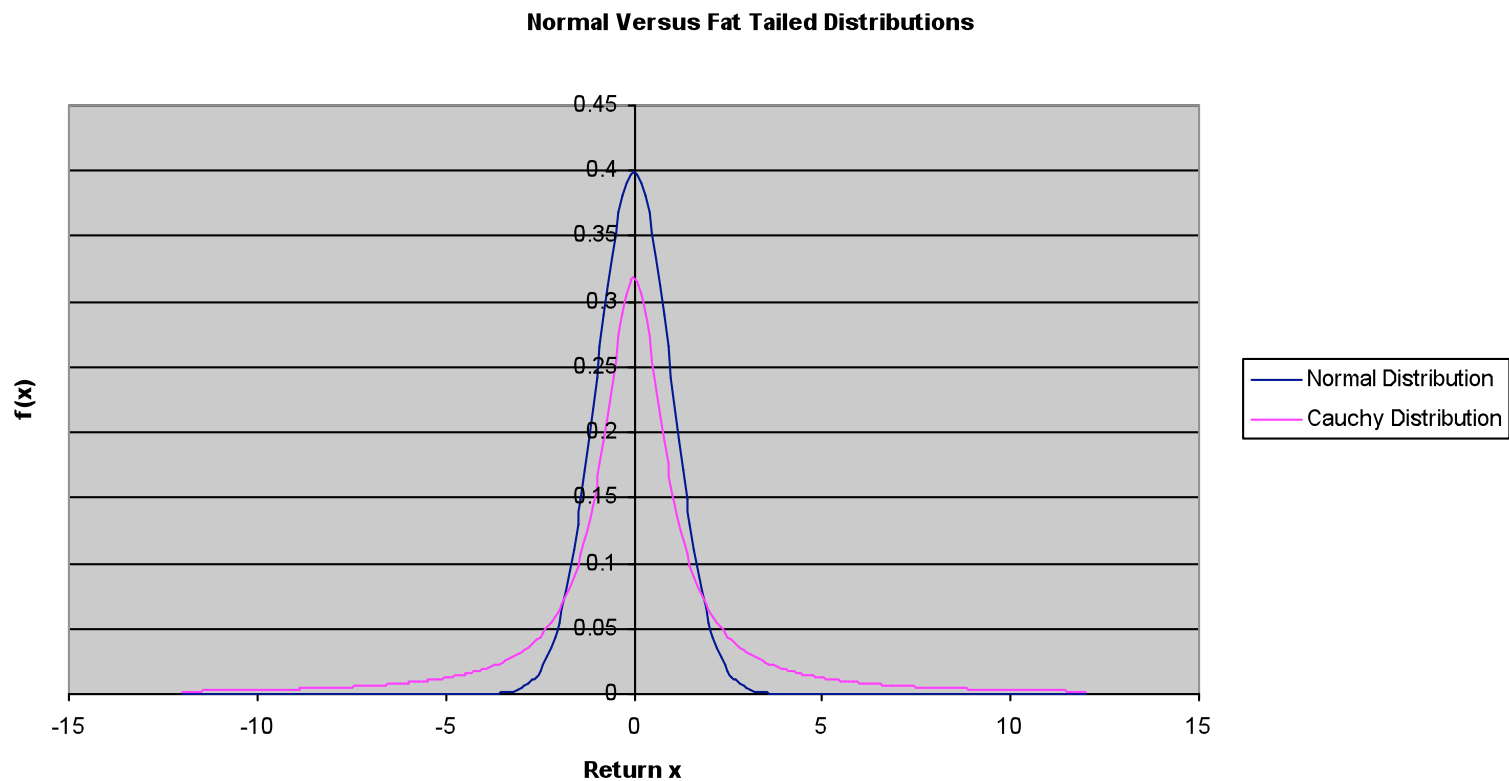
Normal Distribution



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Normal Versus Fat-Tailed



Present Discounted Value (PDV)

- PDV of a dollar in one year = $1/(1+r)$
- PDV of a dollar in n years = $1/(1+r)^n$
- PDV of a stream of payments x_1, \dots, x_n

$$\text{PDV} = \sum_{t=1}^T x_t / (1+r)^t$$

Consol and Annuity Formulas

- Consol pays constant quantity x forever
- Growing consol pays $x(1+g)^{(t-1)}$ in t
- Annuity pays x from time 1 to T

$$\text{Consol PDV} = x / r$$

$$\text{Growing Consol PDV} = x / (r - g)$$

$$\text{Annuity PDV} = x \frac{1 - 1/(1+r)^T}{r}$$