Open Yale courses

Lecture 4: Portfolio Diversification and Supporting Financial Institutions

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Prof. Robert Shiller, Yale University

A Portfolio of a Risky and Riskless Asset

- Put x dollars in risky asset 1, 1-x dollars in the riskless asset earning sure return r_f
- Portfolio expected value $r = xr_1 + (1 - x)r_f$
- Portfolio variance = $x^2 \operatorname{var}(return_1)$
- Portfolio standard deviation σ = $|x|\sigma(return_1)$, $x=(r-r_f)/(r_1-r_f)$
- $\sigma = |\frac{r-r_f}{r_1-r_f}|\sigma(return_1)$

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A Portfolio of Two Risky Assets

- Put x_1 dollars in risky asset 1 and $(1- x_1)$ dollars in risky asset 2 .
- Portfolio expected value $r=x_1r_1+(1-x_1)r_2$
- Portfolio variance =

$$x_1^2 \operatorname{var}(return_1) + (1- x_1)^2 \operatorname{var}(return_2) + 2x_1(1- x_1) \operatorname{cov}(return_1, return_2)$$

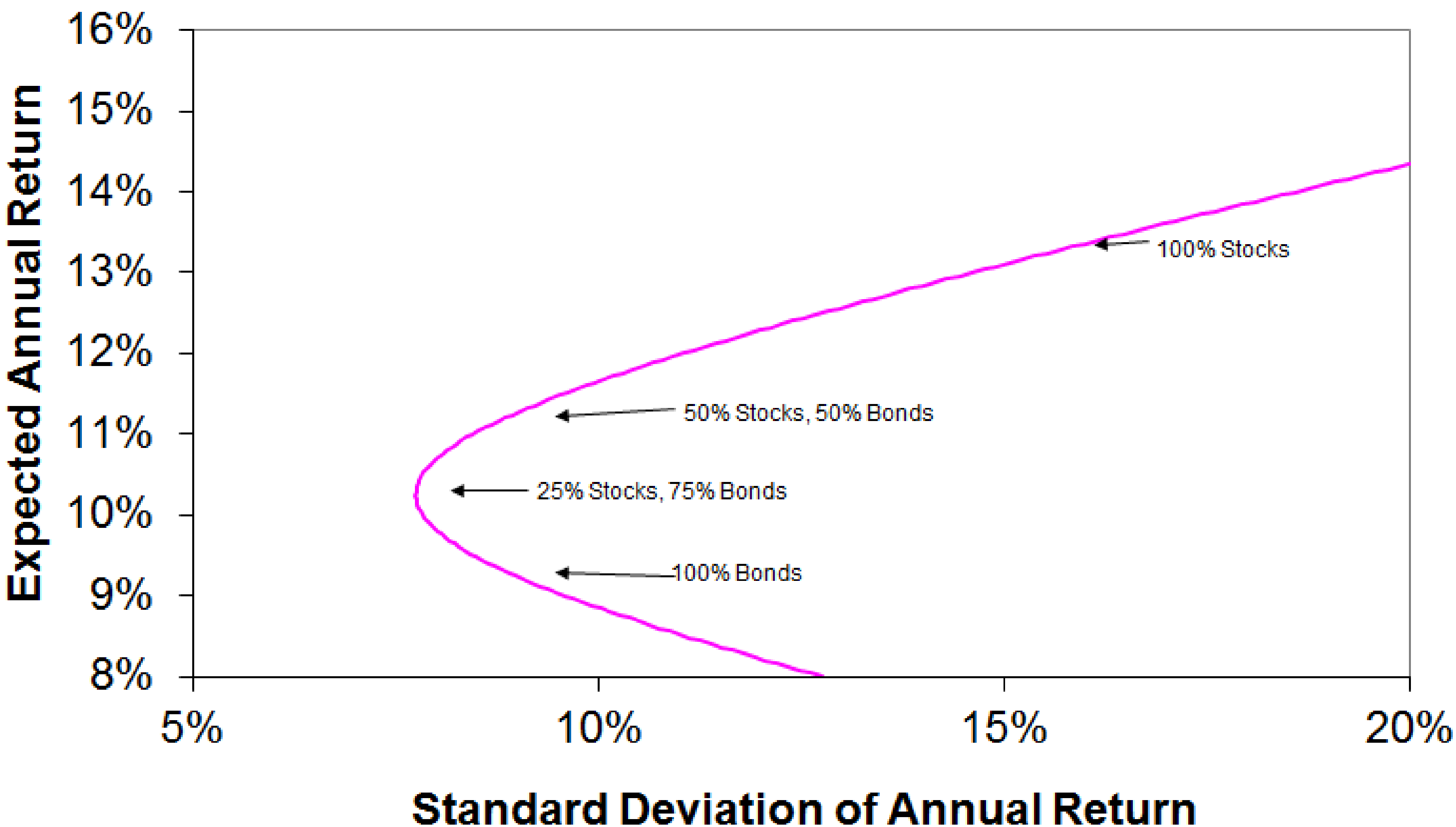
Efficient Portfolio Frontier with Two Assets

- Frontier expresses portfolio standard deviation in terms of portfolio expected return r rather than in terms of x_1.
- $$x_1 = \frac{r - r_2}{r_1 - r_2}$$

$$\sigma^2 = \left(\frac{r - r_2}{r_1 - r_2}\right)^2 \sigma_1^2 + \left(\frac{r_1 - r}{r_1 - r_2}\right)^2 \sigma_2^2$$

$$+ 2\frac{(r - r_2)(r_1 - r)}{(r_1 - r_2)^2}\sigma_{12}$$

Efficient Portfolio Frontier Stocks and Bonds

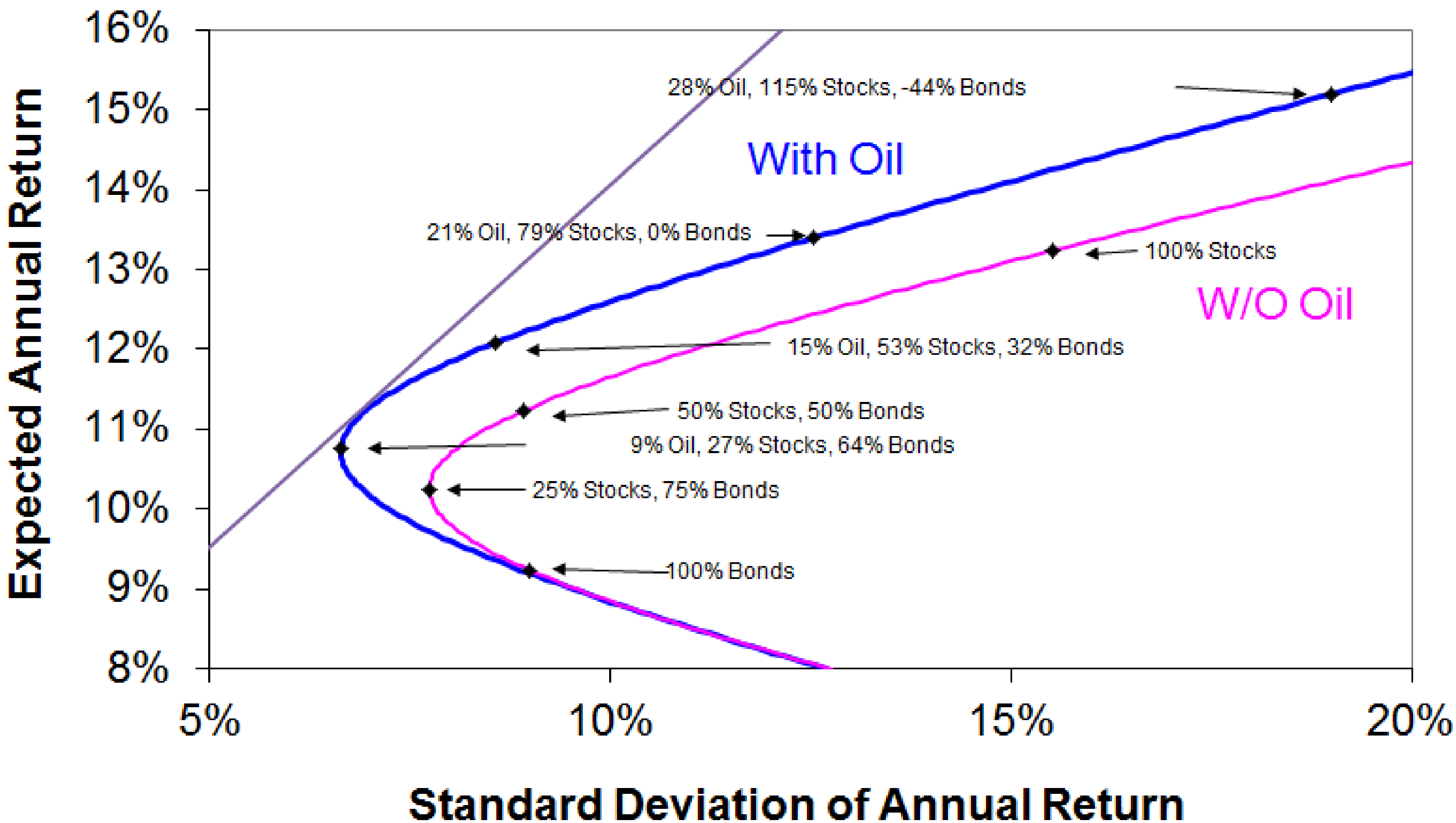


Portfolio Variance, Three Risky Assets

- Portfolio variance =

$$x_1^2 \operatorname{var}(return_1) + x_2^{\,2} \operatorname{var}(return_2) + x_3^2 \operatorname{var}(return_3)$$
$$+ 2x_1x_2 \operatorname{cov}(return_1, return_2) + 2x_1x_3 \operatorname{cov}(return_1, return_3)$$
$$+ 2x_2x_3 \operatorname{cov}(return_2, return_3)$$
$$(\text{where} \sum_{i=1}^{3} x_i = 1)$$

Efficient Portfolio Frontier With and Without Oil



Sharpe Ratio for a Portfolio

$$SharpeRatio = \frac{R(portfolio) - R_f}{\sigma\,(portfolio)}$$

- The Sharpe Ratio is constant along the tangency line
- A portfolio manager is outperforming only if her portfolio has a greater Sharpe ratio

Beta

- The CAPM implies that the expected return on the ith asset is determined from its beta.
- Beta (β_i) is the regression slope coefficient when the return on the ith asset is regressed on the return on the market.
- Fundamental equation of the CAPM:

$$r_i = r_f + \beta_i (r_m - r_f)$$