

Lecture 4: Portfolio Diversification and Supporting Financial Institutions

Economics 252, Spring 2011

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A Portfolio of a Risky and Riskless Asset

- Put x dollars in risky asset 1, $1-x$ dollars in the riskless asset earning sure return r_f
- Portfolio expected value $r = xr_1 + (1-x)r_f$
- Portfolio variance = $x^2 \text{var}(\text{return}_1)$
- Portfolio standard deviation $\sigma = |x|\sigma(\text{return}_1)$, $x = (r - r_f) / (r_1 - r_f)$
- $\sigma = \left| \frac{r - r_f}{r_1 - r_f} \right| \sigma(\text{return}_1)$

A Portfolio of Two Risky Assets

- Put x_1 dollars in risky asset 1 and $(1 - x_1)$ dollars in risky asset 2 .
- Portfolio expected value $r = x_1 r_1 + (1 - x_1) r_2$
- Portfolio variance =

$$x_1^2 \text{var}(\text{return}_1) + (1 - x_1)^2 \text{var}(\text{return}_2) + 2x_1(1 - x_1) \text{cov}(\text{return}_1, \text{return}_2)$$

Efficient Portfolio Frontier with Two Assets

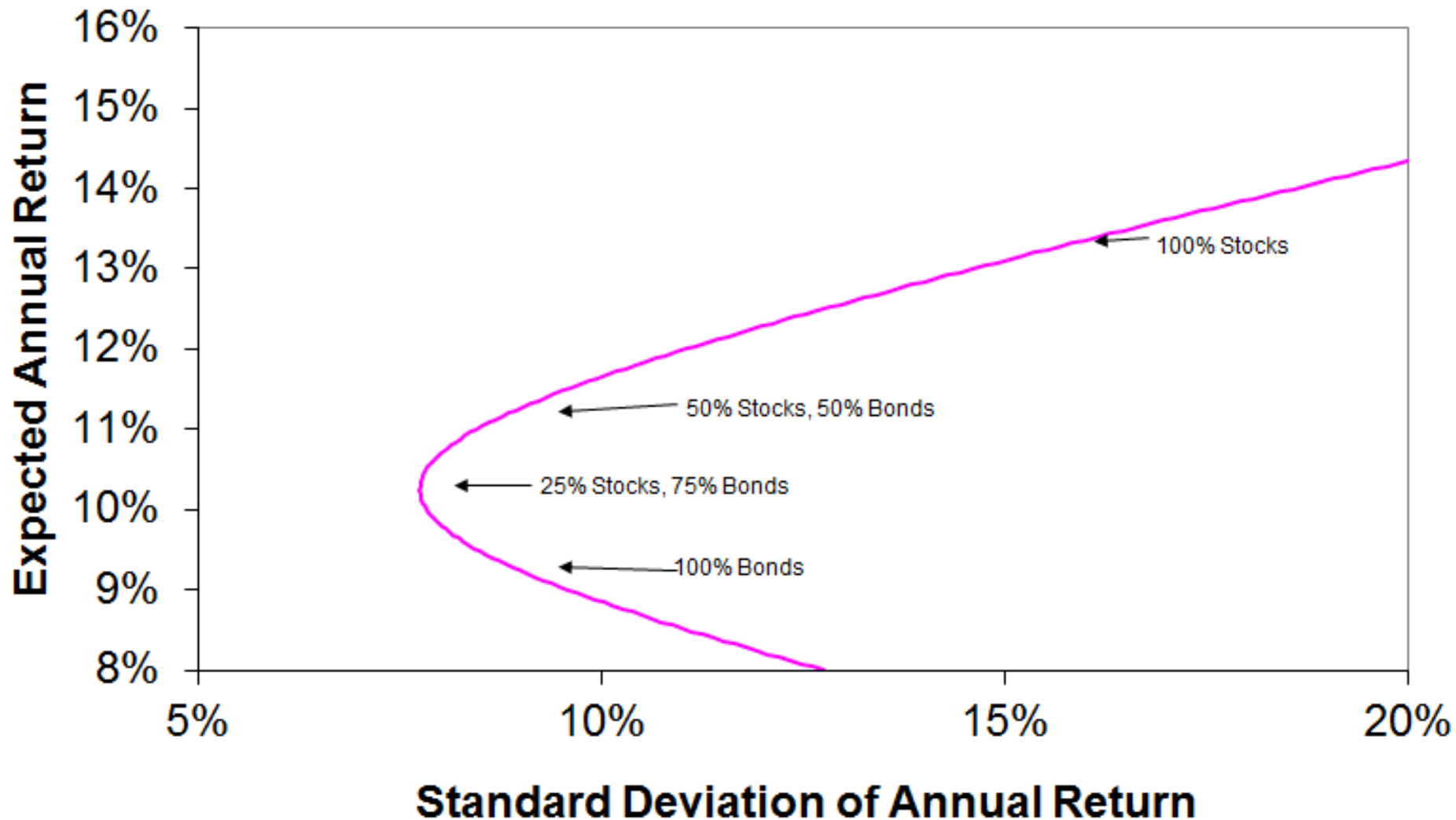
- Frontier expresses portfolio standard deviation in terms of portfolio expected return r rather than in terms of x_1 .

- $$x_1 = \frac{r - r_2}{r_1 - r_2}$$

$$\sigma^2 = \left(\frac{r - r_2}{r_1 - r_2}\right)^2 \sigma_1^2 + \left(\frac{r_1 - r}{r_1 - r_2}\right)^2 \sigma_2^2 + 2 \frac{(r - r_2)(r_1 - r)}{(r_1 - r_2)^2} \sigma_{12}$$

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Efficient Frontier: Frontier Stocks and Bonds



Portfolio Variance, Three Risky Assets

- Portfolio variance =

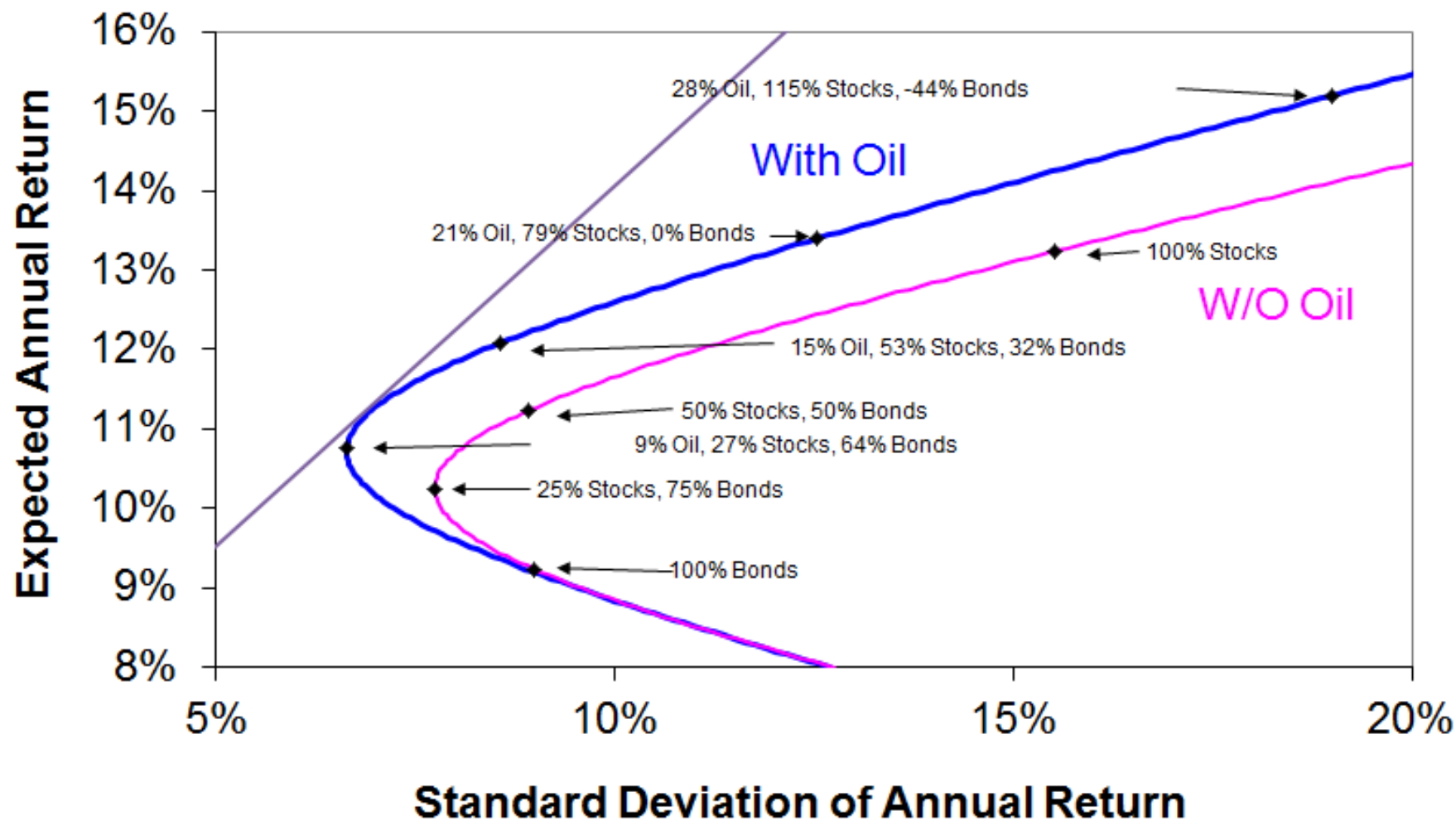
$$\begin{aligned} & x_1^2 \text{var}(\text{return}_1) + x_2^2 \text{var}(\text{return}_2) + x_3^2 \text{var}(\text{return}_3) \\ & + 2x_1x_2 \text{cov}(\text{return}_1, \text{return}_2) + 2x_1x_3 \text{cov}(\text{return}_1, \text{return}_3) \\ & + 2x_2x_3 \text{cov}(\text{return}_2, \text{return}_3) \end{aligned}$$

$$\text{(where } \sum_{i=1}^3 x_i = 1)$$

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Efficient Frontier With and Without Oil



Sharpe Ratio for a Portfolio

$$\text{SharpeRatio} = \frac{R(\text{portfolio}) - R_f}{\sigma(\text{portfolio})}$$

- The Sharpe Ratio is constant along the tangency line
- A portfolio manager is outperforming only if her portfolio has a greater Sharpe ratio

Beta

- The CAPM implies that the expected return on the i th asset is determined from its beta.
- Beta (β_i) is the regression slope coefficient when the return on the i th asset is regressed on the return on the market.
- Fundamental equation of the CAPM:

$$r_i = r_f + \beta_i (r_m - r_f)$$