Lecture 4: Portfolio Diversification and Supporting Financial Institutions

Economics 252, Spring 2011

Prof. Robert Shiller, Yale University
A Portfolio of a Risky and Riskless Asset

- Put $x$ dollars in risky asset 1, $1-x$ dollars in the riskless asset earning sure return $r_f$
- Portfolio expected value $r = xr_1 + (1-x)r_f$
- Portfolio variance $= x^2 \text{var}(\text{return}_1)$
- Portfolio standard deviation $\sigma = |x|\sigma(\text{return}_1)$, $x = (r-r_f)/(r_1-r_f)$
- $\sigma = \left| \frac{r-r_f}{r_1-r_f} \right| \sigma(\text{return}_1)$
A Portfolio of Two Risky Assets

• Put $x_1$ dollars in risky asset 1 and $(1-x_1)$ dollars in risky asset 2.

• Portfolio expected value: $r = x_1 r_1 + (1-x_1) r_2$

• Portfolio variance:

$$x_1^2 \text{var}(\text{return}_1) + (1-x_1)^2 \text{var}(\text{return}_2) + 2x_1(1-x_1) \text{cov}(\text{return}_1, \text{return}_2)$$
Efficient Portfolio Frontier with Two Assets

- Frontier expresses portfolio standard deviation in terms of portfolio expected return $r$ rather than in terms of $x_1$.

- $x_1 = \frac{r - r_2}{r_1 - r_2}$

- $\sigma^2 = \left(\frac{r - r_2}{r_1 - r_2}\right)^2 \sigma_1^2 + \left(\frac{r_1 - r}{r_1 - r_2}\right)^2 \sigma_2^2$

- $+ 2 \frac{(r - r_2)(r_1 - r)}{(r_1 - r_2)^2} \sigma_{12}$
The graph illustrates the relationship between the expected annual return and the standard deviation of annual return for different asset allocations between stocks and bonds. The x-axis represents the standard deviation of annual return ranging from 5% to 20%, while the y-axis represents the expected annual return ranging from 8% to 16%.

- The curve shows that as the percentage of stocks increases, the expected return increases but so does the risk (standard deviation).
- At 100% stocks, the expected return is highest but also the risk is highest.
- At 100% bonds, the expected return is lowest but the risk is also lowest.
- A 50% stocks, 50% bonds allocation offers a balance between return and risk.
Portfolio variance, Three Risky Assets

- Portfolio variance =

\[ x_1^2 \text{var}(\text{return}_1) + x_2^2 \text{var}(\text{return}_2) + x_3^2 \text{var}(\text{return}_3) + 2x_1x_2 \text{cov}(\text{return}_1, \text{return}_2) + 2x_1x_3 \text{cov}(\text{return}_1, \text{return}_3) + 2x_2x_3 \text{cov}(\text{return}_2, \text{return}_3) \]

(where \( \sum_{i=1}^{3} x_i = 1 \))
Efficient Portfolio Frontier With and Without Oil

- 28% Oil, 115% Stocks, -44% Bonds
- 21% Oil, 79% Stocks, 0% Bonds
- 15% Oil, 53% Stocks, 32% Bonds
- 50% Stocks, 50% Bonds
- 9% Oil, 27% Stocks, 64% Bonds
- 25% Stocks, 75% Bonds
- 100% Bonds
- 100% Stocks

- Expected Annual Return
- Standard Deviation of Annual Return

© Yale University 2012. Most of the lectures and course material within Open Yale Courses are licensed under a Creative Commons Attribution-Noncommercial-Share Alike 3.0 license. Unless explicitly set forth in the applicable Credits section of a lecture and/or content, all third-party content is not covered under the Creative Commons license. Please consult the Open Yale Courses Terms of Use for limitations and further explanations on the application of the Creative Commons license.
Sharpe Ratio for a Portfolio

\[ \text{Sharpe Ratio} = \frac{R(\text{portfolio}) - R_f}{\sigma(\text{portfolio})} \]

- The Sharpe Ratio is constant along the tangency line.
- A portfolio manager is outperforming only if her portfolio has a greater Sharpe ratio.
Beta

• The CAPM implies that the expected return on the ith asset is determined from its beta.
• Beta ($\beta_i$) is the regression slope coefficient when the return on the ith asset is regressed on the return on the market.
• Fundamental equation of the CAPM:

$$r_i = r_f + \beta_i (r_m - r_f)$$