Question 1

(a) How are futures and options different in terms of the risks they allow investors to protect against?

(b) Consider a corn farmer, who will harvest 20,000 bushels of corn in 3 months. Using the proceeds from the sale of his corn, he plans to pay back a loan in the amount of $140,000 that is due in 3 months. The price of corn today is $7 per bushel. Yet, the farmer receives some news indicating that the price of corn might substantially change within the next 3 months. The farmer gets nervous, as an adverse price change in corn price might render him unable to make his payments.

Which of the following alternatives would you recommend to him?

- Buy futures contracts for 20,000 bushels with a futures price of $7 and delivery in 3 months.
- Sell futures contracts as they are outlined in the previous alternative.
- Buy European put options for 20,000 bushels with a strike price of $2 per bushel.

Explain your answer by describing his profit in various states of the world.
CONTINUATION OF QUESTION 1.

(c) Consider a manager, who wants to expand his small business. The manager predicts that stock ABC is currently overpriced and that its price will decline within the next 6 months. ABC stock sells for $40 today and the manager believes that the stock price will be $25 in 6 months.

He is wondering, whether he can use his insight on the future price movements of company ABC to raise some funding for his new project. While he is very much excited about the new project and thus is willing to take some risks to generate funding for it, he does not want to jeopardize his ongoing project by incurring losses because of speculation on ABC.

Which of the following alternatives would you recommend to him?

- Buy European call options with a strike price of $30 maturing in 6 months.
- Buy futures contracts written on ABC with a futures price of $30 and delivery in 6 months.
- Buy European put options with a strike price of $30 maturing in 6 months?

Explain your answer by describing his profit in various states of the world.
Question 2

Assume that all call and put options mentioned below are European style, have the same maturity date, and are written on stock XYZ.

Moreover, ignore any discounting between the date at which an option is purchased and the date at which it matures.

(a) Draw the payoff and profit at the maturity date for a call option $C_1$ on XYZ stock from the perspective of the buyer. Suppose that the strike price is $40$ and that the call option costs $8$.

(b) Draw the payoff and profit at the maturity date for a put option $P_1$ on XYZ stock from the perspective of the buyer. Suppose that the strike price is $40$ and that the put option costs $12$.

(c) Suppose an investor purchases a “straddle.” That is, he buys one unit of the call in (a) and one unit of the put in (b).

Draw the payoff and profit of this portfolio at the maturity date.

Judging from your graph, why do you think he might want to construct this portfolio?

(d) Suppose an investor constructs a “bull spread.” That is, he buys one unit of the call $C_1$ from part (a) and sells one call option $C_2$ on XYZ stock with a strike price of $50$, which has a cost of $5$.

Draw the payoff and profit of this portfolio at the maturity date.

Judging from your graph, why do you think he might want to construct this portfolio?
(e) Suppose an investor creates a "butterfly". That is, he buys one unit of the call $C_1$ from part (a) and buys unit of the call $C_2$ from part (d). In addition, he writes two call options $C_3$ on XYZ stock with a strike price of $45, which cost $6 each.

Draw the payoff and profit of this portfolio at the maturity date.

Judging from your graph, why do you think he might want to construct this portfolio?
Question 3

The price of RJS stock is currently $100. The stock price in each of the next two years will either increase by a factor of $u=1.5$ or decrease by a factor of $d=0.5$, following the notation in class.
(Stock increases and decreases in different years are independent from each other.)

The annual risk-free interest rate for the next two years is constant at 25%. RJS stock does not pay any dividends.

Consider a European call option $C_1$ written on RJS stock with a strike price of $30$, which matures in 1 year.

(a) What is the hedge ratio for this call option? What is the meaning of the hedge ratio?

(b) What is the price of the call option $C_1$ today?

Now, consider another European call option $C_2$ written RJS stock with a strike price of $30$, which matures in 2 years.

(c) What is the price of $C_2$ a year from now after the price has gone down once? What is the price of $C_2$ today?

(d) How did the increase in maturity influence the call price today?

(e) Using your answer in part (c) and the put-call parity, calculate the price of a 2-year put with the same strike price as $C_2$ a year from now after the price has gone down once. Additionally, what is the price of this put option today?
Question 4

Assume that the risk-free interest rate is 20% per year. Suppose that the current price of the underlying stock is $200 and that the standard deviation of the return on the stock (also known as the volatility) is 0.5.

(a) Use the Black-Scholes Option Pricing formula to find the value of a 4-year call option with a strike price of $120.
   (Hint: NORMSDIST(d) in Excel gives you the probability that a random variable with a standard normal probability distribution is less than (or equal to) d.)

(b) Calculate the value of a 4-year put option with a strike price of $120 according to the Black-Scholes Option Pricing formula.
   (Hint: Use the continuous-time version of the put-call parity)

(c) Suppose that the same terms for the options apply, but suppose that the volatility decreases to 0.4. Compute the new values of the call and the put option according to the Black-Scholes Option Pricing formula.
   How do the new values compare to the values computed in parts (a) and (b)? Provide some intuition for your findings.

(d) Suppose that the stock's volatility is back to 0.5, but suppose that the strike price for both the 4-year call and the 4-year put option decreases to $100. Re-compute the values of the call and the put option according to the Black-Scholes Option Pricing formula.
   How do the new values compare to the values computed in parts (a) and (b)? Provide some intuition for your findings.