Econ 252 - Financial Markets

Spring 2011

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Problem Set 2

Question 1

Consider the following three assets:

- Asset A's expected return is 5% and return standard deviation is 25%.
- Asset B's expected return is 8% and return standard deviation is 32%.
- Asset C is a risk-free asset with 2% return.

The correlation between assets A and B is -0.3.

(a) Constructing a portfolio from assets A and B such that the expected return of the portfolio equals 7%, find the portfolio weights of assets A and B and compute the return standard deviation of the portfolio.

(b) Constructing a portfolio from assets A and B such that the expected return of the portfolio equals 3%, find the portfolio weights of assets A and B and compute the return standard deviation of the portfolio.

(c) Constructing a portfolio from assets A and C such that the expected return of the portfolio equals 2.5%, find the portfolio weights of assets A and C and compute the return standard deviation of the portfolio.

(d) Constructing a portfolio from assets A and C such that the expected return of the portfolio equals 10%, find the portfolio weights of assets A and C and compute the return standard deviation of the portfolio.

Question 2

Consider the following two assets:

- Asset A's expected return is 4% and return standard deviation is 42%.
- Asset B's expected return is 1.5% and return standard deviation is 24%.

The correlation between assets A and B is 0.1.

(a) Compute the expected return and the return standard deviation for a portfolio putting weight w on asset A and weight 1-w on asset B for w=-0.5, w=0.3, w=0.8, w=1.3.

You now have six points on a portfolio frontier involving assets A and B.

(b) Carefully draw a sketch of the portfolio frontier in a μ-σ-diagram. Mark the three regions in which w is negative, between 0 and 1, and bigger than 1. Additionally, indicate the minimum variance portfolio.

Assume that the upper part of the portfolio frontier that you have just drawn represents the Efficient Portfolio Frontier of some set of assets that will not be specified any further.

(c) Draw the tangency line associated with a risk-free return of 1%. Indicate the Tangency/Market Portfolio.

(d) Draw another tangency line associated with a risk-free return of 0%. Again, indicate the Tangency/Market Portfolio.

(e) Assume you want to minimize return standard deviation for given levels of expected return. If you need to construct a portfolio with expected return 5%, would you rather want to use the risk-free rate in part (c) or the one in part (d).

Question 3

Consider a Market Portfolio with 12% expected return and 20% return standard deviation.

(a) If the Sharpe ratio of the market portfolio is 0.5, what is the risk-free rate of return?

(b) Consider the following three portfolios on the tangency line:

- Portfolio 1 has 6.3% expected return.
- Portfolio 2 has 8.25% expected return.
- Portfolio 3 has 17.8% expected return.

Find the return standard deviation of each of the three portfolios.

(Hint: Use the fact that all portfolios are located on the tangency line.)

In the context of the Mutual Fund Theorem, one is not able to specify the exact portfolio that an agent will choose on the Tangency Line, as long as one does not know the exact tradeoff between expected return and return standard deviation that this agent faces.
In the following, an explicit tradeoff for an agent will be specified. This will allow finding the exact portfolio on the Tangency Line that will be optimal for this agent.

So, assume that the utility that an agent derives from any portfolio is given by

$$u(\text{portfolio}) = \mu - 2\sigma^2.$$

That is, the utility of the portfolio is its expected return minus twice its variance.

(c) Which of the three portfolios from part (b) will the agent with the above utility function choose?

In fact, your answer is the portfolio that the agent will choose among all possible portfolios consisting of the risk-free asset and the market portfolio.

QUESTION 3 CONTINUES ON THE NEXT PAGE.

CONTINUATION OF QUESTION 3.

(d) (This part is optional and not relevant for any of the exams.)

Formally derive the optimality of the portfolio from part (c).

(Hint: Derive expressions, in terms of the weight w on the risk-free asset, for the expected return and the return variance of an arbitrary portfolio consisting of the risk-free asset and the Market Portfolio. Plug these expressions into the utility-function above and optimize with respect to w. Check the SOC (second-order condition) to make sure that you have found a maximum.)

Question 4

Consider the following year-end prices of a hypothetical market index:

Year	Price
2004	100
2005	110
2006	104.5
2007	106.59
2008	106.59
2009	110.85
2010	108.63

(a) Compute the expected (annual) return of the market index as the arithmetic average of the annual returns of the market index.

Assume that the risk-free rate equals 0.75% and use the above market index as the Market Portfolio.

(b) Assuming that the CAPM holds, what is the expected return of asset A whose beta equals 0.8?

(c) Again assuming that the CAPM holds, what is the expected return of asset B whose beta equals 3?

(d) What, according to the CAPM, is the beta of an asset C whose expected return equals 0.75% (that is, the risk-free rate in this problem)?