Question 1

A lottery sells 1 million tickets.

One of those tickets wins the grand prize of $1 million, 100 tickets win 1st place prizes of $10,000, and 10,000 tickets win prizes of $1.

(a) What is the expected value of winnings from a single lottery ticket?

(b) What is the variance of the winnings from a single lottery ticket?

(c) If lottery tickets cost $4, should you buy one? Why? What if they cost $1?
Question 2

Consider three bonds, each promising to pay $100 in 10 years.

The first bond is a U.S. bond (think about it as a U.S. Treasury bond) that always pays.

The other two are state bonds for New York and California, which may pay the full $100 dollars, or may renegotiate and only pay $80, or may default and only pay $20 in bankruptcy. The corresponding probabilities of these events occurring are:

<table>
<thead>
<tr>
<th></th>
<th>CA pays $100</th>
<th>CA pays $80</th>
<th>CA pays $20</th>
</tr>
</thead>
<tbody>
<tr>
<td>NY pays $100</td>
<td>.3</td>
<td>.15</td>
<td>.05</td>
</tr>
<tr>
<td>NY pays $80</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
</tr>
<tr>
<td>NY pays $20</td>
<td>.05</td>
<td>.05</td>
<td>.1</td>
</tr>
</tbody>
</table>

(a) What is the expected payment of each of the three bonds in 10 years?

(b) What is the variance of the payments in 10 years of each of the three bonds?

(c) What is the standard deviation of the payments in 10 years of each of the three bonds?

(d) What is the covariance of the payments in 10 years of the New York bond and the California bond?

(e) What’s the correlation of the payments in 10 years of the New York bond and the California bond?

(f) What’s the expectation and variance of the payments in 10 years for a portfolio made up of 1/3 of the U.S. bond, 1/3 of the NY bond and 1/3 of the CA bond?
Question 3

Consider the following two assets:

- Asset A’s expected return is 10% and return standard deviation is 20%.
- Asset B’s expected return is 5% and return standard deviation is 15%.

The correlation between assets A and B is 0.5.

The table below indicates the expected return and the return standard deviation for portfolios that put weight \( w \) on asset A and weight \( 1-w \) on asset B.

<table>
<thead>
<tr>
<th>Weight</th>
<th>Expected Return</th>
<th>Return Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w=0.75 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w=0.50 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w=0.25 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Complete the above table.

(b) Instead of a correlation of 0.5 between assets A and B, consider a correlation of -0.5 and re-compute the above table.